Geometry during compression

LS-DYNA Simulation of liner implosion

- Eulerian VOF calculation
- Johnson-Cook model for aluminum 6061 liner (blue)
- Jones-Wilkins-Lee equation of state for chemical driver (red)
- Parameters tuned based on field tests

MHD time-dependent mesh is generated from smoothed LS-DYNA result
Cross section of SPECTOR PCS experiment

- Shaft current (white arrows) provides toroidal field
- Mirnov probes (colored dots) measure poloidal and toroidal fields
- Inner electrode (Center Shaft) is shaped for 4:1 compression
MHD Simulation with Versatile Advection Code (VAC)

Shock capturing Eulerian Finite Volume code by Gábor Tóth.

In-house modifications:
- Improvements for strong toroidal fields (e.g., slope-limiting $rB_\phi$ instead of $B_\phi$)
- Coupling MHD to external circuit models
- Independent ion and electron temperatures
- Classical parallel heat transport

Transport:
- Spitzer temperature dependent resistivity
- Various models for radial heat transport, $\chi$
- Constant viscosity for simplicity

\[
\frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \mathbf{v}) \\
\frac{\partial (\rho \mathbf{v})}{\partial t} = - \nabla \cdot (\rho \mathbf{v} - \mu_0^{-1} \mathbf{B} \mathbf{B}) - \nabla p_* + \nabla \cdot \mathbf{Y} \\
\frac{\partial \mathbf{B}}{\partial t} = - \nabla \cdot (\mathbf{v} \mathbf{B} - \mathbf{B} \mathbf{v}) - \nabla \times \mathbf{E'} + e_\infty f(r, z) V_{\text{kin}}(t) \\
\frac{\partial e_{\text{th,e}}}{\partial t} = - \nabla \cdot (\mathbf{v} e_{\text{th,e}}) - (\gamma - 1) e_{\text{th,e}} \nabla \cdot \mathbf{v} + G_{e_i} - \nabla \cdot \mathbf{q}_e + \mathbf{E'} \cdot \mathbf{J} \\
\frac{\partial e_{\text{th,i}}}{\partial t} = - \nabla \cdot (\mathbf{v} e_{\text{th,i}}) - (\gamma - 1) e_{\text{th,i}} \nabla \cdot \mathbf{v} - G_{e_i} - \nabla \cdot \mathbf{q}_i + \mathbf{Y} \\
\frac{\partial q_{\|,e}}{\partial t} = - \nabla \cdot (\mathbf{v} q_{\|,e}) - \frac{5}{2} \frac{k T_e}{m_e |\mathbf{B}|} \nabla (k T_e) - q_{\|,e} \tau_{q,e} \\
\frac{\partial q_{\|,i}}{\partial t} = - \nabla \cdot (\mathbf{v} q_{\|,i}) - \frac{5}{2} \frac{k T_i}{m_i |\mathbf{B}|} \nabla (k T_i) - q_{\|,i} \tau_{q,i} \\
\]
MHD simulation uses quasi-static compression

Compression is much slower than plasma dynamics

Time scales and velocities:

<table>
<thead>
<tr>
<th>Physics</th>
<th>Time scale</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistive decay</td>
<td>$\tau_{\text{res}} \approx 1000 \mu s$</td>
<td></td>
</tr>
<tr>
<td>Compression</td>
<td>$\tau_{\text{compr}} \approx 130 \mu s$</td>
<td>$v_{\text{compr}} \approx 1.5 \times 10^3 \text{ m/s}$</td>
</tr>
<tr>
<td>Plasma sound</td>
<td>$\tau_s \approx 1.2 \mu s$</td>
<td>$c_s \approx 10^5 \text{ m/s}$</td>
</tr>
<tr>
<td>Alfvén wave</td>
<td>$\tau_A \approx 0.1 \mu s$</td>
<td>$v_A \approx 10^6 \text{ m/s}$</td>
</tr>
</tbody>
</table>

Time scale ordering:

$\tau_{\text{res}} > \tau_{\text{compr}} \gg \tau_s > \tau_A$

$\therefore$ Use quasi-static approximation to implement compression.
Compression step algorithm

Quasi-static approximation: every 100 time steps (1-10 ns)
First: transform physical quantities to compression invariants:

<table>
<thead>
<tr>
<th>Invariant</th>
<th>Conserved quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{g}\rho$</td>
<td>mass</td>
</tr>
<tr>
<td>$\sqrt{g}\rho\nu_i$</td>
<td>angular momentum</td>
</tr>
<tr>
<td>$\sqrt{g}B^i$</td>
<td>magnetic flux</td>
</tr>
<tr>
<td>$p/\rho^{5/3}$</td>
<td>specific entropy</td>
</tr>
</tbody>
</table>

Tensor components are with respect to logical coordinates.

Next: Update mesh geometry.

Finally: Transform back to physical quantities using new coordinate transform.
Initial Conditions

Grad-Shafranov equilibrium

Density = $7 \times 10^{19} \text{ m}^{-3}$

Temperature = 200 eV (flux function)

Poloidal flux of CT = 16.7 mWb (matches experimental Mirnov signals)

Plasma current = 206 kA

Shaft current = 260 kA
Ramping shaft current stabilizes CT

Constant shaft current:
\[ I_{\text{shaft}}(t) = I_0 \]

Ramped shaft current:
\[ I_{\text{shaft}}(t) \propto \frac{1}{r(t)} \]

\[ \lambda = \frac{\mu_0 \vec{J} \cdot \vec{B}}{\vec{B} \cdot \vec{B}} \]
Compression ratio \( \frac{R_0}{R(t)} \)

PCS 13 and Simulation: Toroidal Field
Neither ramps shaft current

Thin squiggly lines: PCS 13 experimental compression
Thicker lines: sr189 3D simulation

\[ \mu_0 I = 2\pi r B_\phi \]
Compression ratio = \( \frac{R_0}{R(t)} \)

Simulation & Experimental Compression
Simulation captured instability
Thin lines: PCS 13 compression experiment
Thicker lines: sr189 3D simulation

Simulation Compression
Shaft current suppressed instability
Dashed lines: sr189 3D without shaft current ramp
Normal lines: sr191 3D with shaft current ramp
Compression ratio = \( \frac{R_0}{R(t)} \)

\[ \mu_0 I = 2\pi r B_{\phi} \]
PCS 14 Compression: Uniform Chi of 15 Experiment and Simulation

Bold Lines: sr649 2D Simulation
Thin Lines: PCS 14 Experiment

100 μs Simulation Lead In  Wall Move  Slapdown  AXUV Cutoff
Numerical Experiment

- Assume edge cooling at 920 \( \mu s \) (1.7x)
- Switched to non-uniform \( \chi \) profile
- This mimics either:
  - Increased transport
  - Particulate cooling
Compression heating of plasma

Electron temperature during compression with and without chi step

\[ \chi_{\text{wall}} = 1000 \text{ mm}^2/\mu\text{s} \]

at time=920 \( \mu \text{s} \)

- e\(^{-}\) Temp [eV] - Step \( \chi_{\text{wall}} = 1000 \)
- e\(^{-}\) Temp [eV] - Constant \( \chi = 15 \)

wall move
time=805\( \mu \text{s} \)

Pressure during compression with and without chi step

\[ \chi_{\text{wall}} = 1000 \text{ mm}^2/\mu\text{s} \]

at time=920 \( \mu \text{s} \)

- P [Pa] - Step \( \chi_{\text{wall}} = 1000 \)
- P [Pa] - Constant \( \chi = 15 \)

wall move
time=805\( \mu \text{s} \)
Summary

**Shaft current ramp**
- MHD simulations showed stabilizing effect
- Motivated inclusion in PCS14 experiment
- Compression was stable at least to $R_0/R = 2.5x$

**Modeling PCS14**
- MHD simulation initialized to conditions of PCS14
- Matches decay of plasma current prior to compression
- Matches compression increase of plasma current until a compression ratio of about 1.7x, then experiment falls below simulation.

**Numerical experiment**
- Assume edge cooling, model by increased edge transport
- Mirnov signals in better agreement with experiment
- Core temperature, pressure remain relatively high

**Simulation prospects for PCS15**
- AXUV diode x-ray thermometers
- Additional Mirnov probe for better initialization
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