

## Introduction

Here we present the analyses of helicity balance for the SPECTOR (SPHERical Compact TORoid) devices at General Fusion. SPECTOR devices are the reduced-scale plasma injectors at General Fusion designed to enable compression of stable, spherical magnetized plasmas. The toroidal magnetic field is applied by driving a current through the central shaft prior to the ignition of the plasma.

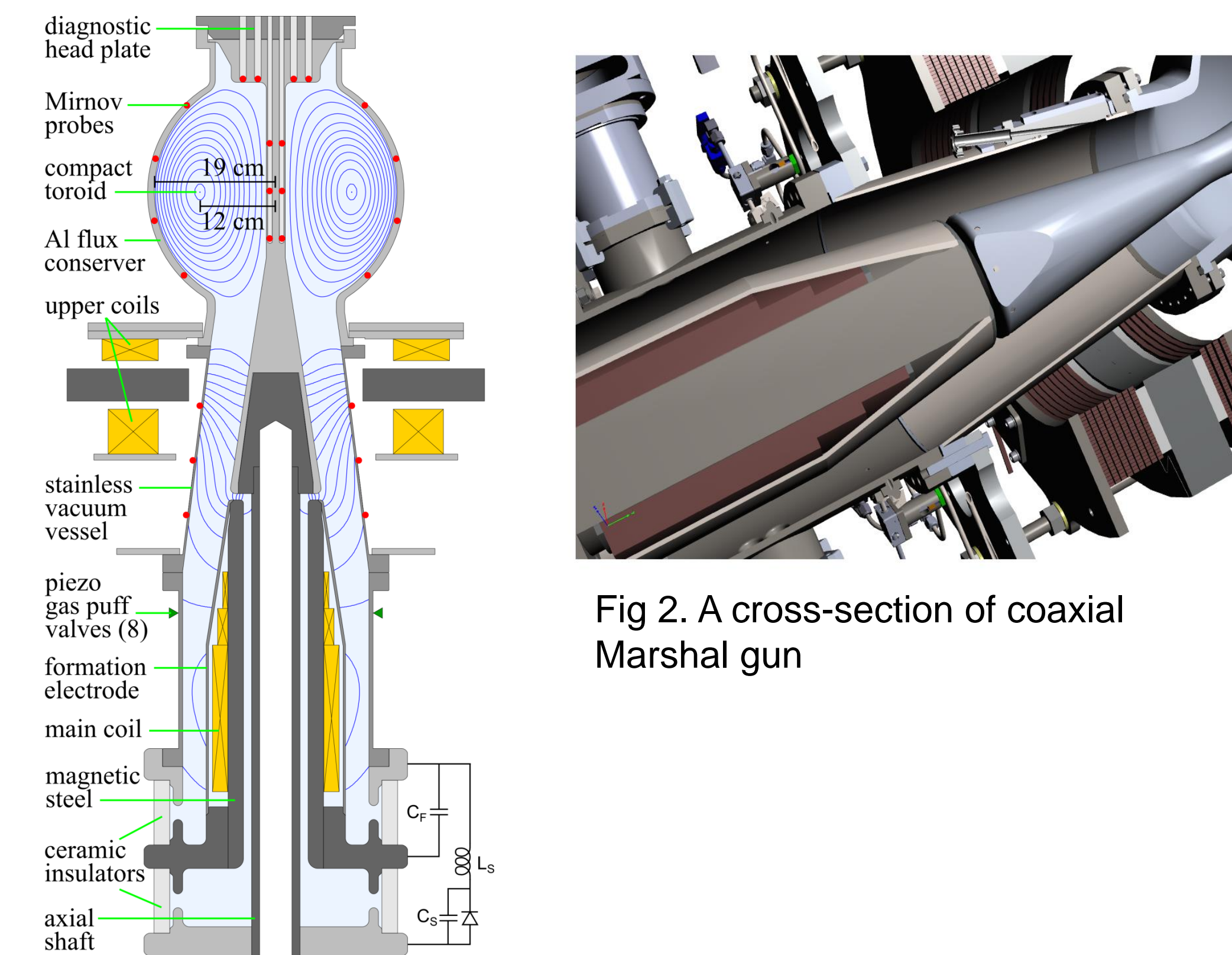


Fig 2. A cross-section of coaxial Marshall gun

Fig 1. A schematic diagram of SPECTOR

- A spherical tokamak with Rinner = 3 cm, Router=19 cm
- Current through axial shaft is 0.5 MA
- Plasma density is  $(1 - 2) \times 10^{14} \text{ cm}^{-3}$
- $T_e$  at plasma center is 350 – 450 eV
- Toroidal magnetic field is 0.5 T at plasma center
- Plasma current of 300-800 kA is induced using coaxial helicity injection
- Coaxial Marshall gun generates 80  $\mu\text{s}$  long formation pulses of current up to 850 kA
- Magnetic gun flux is 10-20 mWb.
- Plasma current is not sustained and resistively decays for 1.5-1.9 ms
- The inner surfaces of the gun and inner wall of the flux conserver are coated with plasma-sprayed tungsten, the outer wall is bare aluminum.

### Diagnostics:

- 41 magnetic probes on the inner and outer walls of the plasma vessel and in the Marshall gun
- Thomson scattering: YAG laser generates 1 pulse per plasma shot 6 radially resolved points are available
- Three-chord FIR heterodyne interferometer with two 118.8 $\mu\text{m}$  methanol cavities
- Three-chord FIR polarimeter for measuring Faraday rotation
- SXR diagnostic using 2 filtered AXUV diodes
- Visible and VUV spectroscopy
- Ion-Doppler spectroscopy

## Helicity Balance

Following [1], the helicity balance in a plasma produced by a gun is given by:

$$\frac{dK}{dt} = 2 V_g \psi_g - 2 \int_{sheath} \mathbf{E} \cdot \mathbf{B} d^3r - 2 \int_{core} \mathbf{E} \cdot \mathbf{B} d^3r \quad (1)$$

where  $V_g$  and  $\psi_g$  are the gun voltage and the poloidal flux, respectively, while the integrals take into account the dissipation in the plasma and in the sheath surrounding the plasma.

Eq. (1) can be rewritten as:

$$\frac{dK}{dt} = 2 (V_g - V_{sheath}) \psi_g - 2 \int_{core-sheath} \mathbf{E} \cdot \mathbf{B} d^3r \quad (2)$$

where  $V_{sheath}$  is the voltage drop across the sheath. The sheath is a region of open field lines that surrounds the well-confined plasma region with closed field lines. Note that for  $V_g > V_{sheath}$  the sheath potential terms can be neglected in the balance, since  $V_g$  is about 6-8 kV during the sustainment phase in SPECTOR.

The plasma dissipation integrand can be written in a simple way as:

$$\mathbf{E} \cdot \mathbf{B} = \eta_{||} \mathbf{J} \cdot \mathbf{B} \quad (3)$$

Therefore, only the parallel current contributes to the core dissipation.

## Constraints Deduced From Taylor State

Taylor's plasma relaxation has an important role in systems like spheromaks, but when there is a strong external toroidal magnetic field, the applicability of Taylor's paradigm is more uncertain. However, assuming such a theory can be applied [2], the following eigenvalues can be obtained for the SPECTOR geometric configuration ( $a=0.014 \text{ m}$ ,  $b=0.19 \text{ m}$ ,  $L=0.4 \text{ m}$ ):  $g_{1,1}=5$ , and  $\lambda_{eig}=5/b=26 \text{ m}^{-1}$ . As discussed in [3], a gun can form a plasma if the following requirement is met:

$$\lambda_{gun} = \frac{\mu_0 I_{gun}}{\psi_{gun}} > \lambda_{eig}$$

With a poloidal flux of 14 mWb and a gun current of 600 kA, the typical values in SPECTOR for  $\lambda_{gun}$  are about 45-50  $\text{m}^{-1}$ . So, this condition is satisfied.

Another important condition discussed in [4], which guarantees the transport of helicity from the gun to the plasma core, is:

$$\lambda_{gun} > \frac{\mu_0 I_{tor}}{\Phi} = \lambda_{tok}$$

where  $I_{tor}$  is the toroidal plasma current and  $\Phi$  is the toroidal flux. Assuming a toroidal field of 0.9 T and a toroidal current of 300 kA,  $\lambda_{tok}=3.5 \text{ m}^{-1}$ , the above condition is also very well satisfied.

Unfortunately, the efficiency of helicity injection  $\epsilon = \lambda_{tok}/\lambda_{gun}$ , as defined in [4], is very low in SPECTOR compared to the efficiency of inductive current drive in tokamaks using an ohmic transformer.

As discussed in [5], plasmas are expected to have a hollow current profile when  $\lambda_{gun} > \lambda_{eig}$ , while peaked profiles are expected when  $\lambda_{gun} < \lambda_{eig}$ . Since  $\lambda_{gun}$  is slightly higher than  $\lambda_{eig}$  in SPECTOR, we expected to form plasmas with flat to slightly hollow plasma current profiles.

## Comparison With Experimental Data

Following [8], the helicity balance equation can be rewritten in the following way:

$$\frac{dK}{dt} = 2 V_g \psi_g - \frac{K}{\tau_k} \quad (4)$$

where the helicity rate of change is the sum of the rate of gun generated helicity (1<sup>st</sup> term) and the rate of CT helicity decay (2<sup>nd</sup> term), with  $\tau_k$  as the helicity confinement time of the system.

The product  $V_g \psi_g$  is based on the real Spector data of the applied poloidal gun flux of 14.8 mWb and the voltage measured between the electrodes of the gun, is shown in Fig.1,

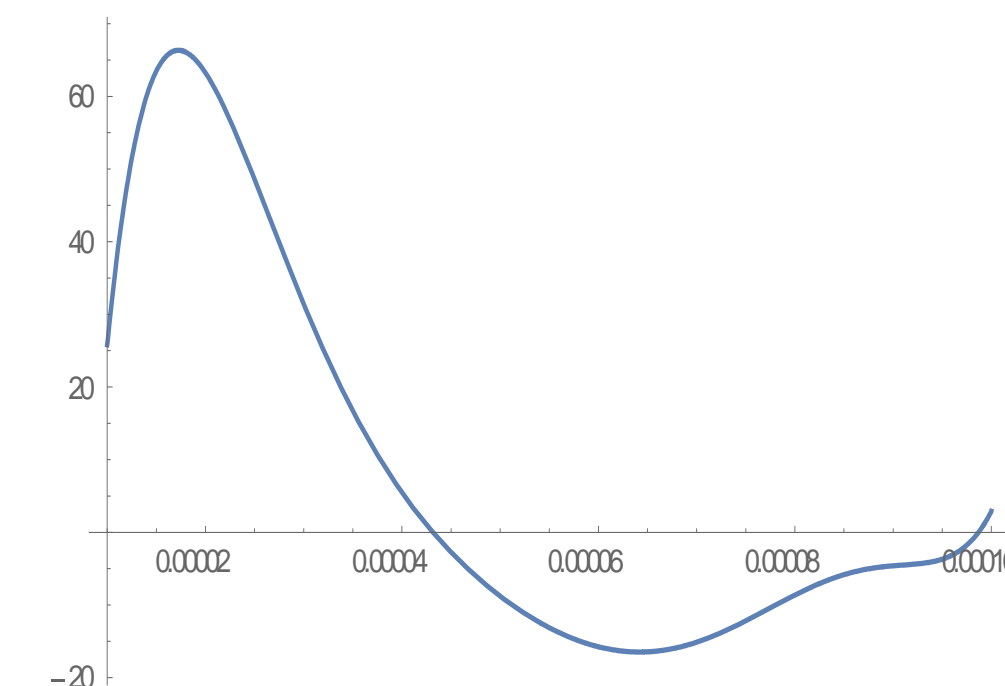


Fig 3. Experimental  $V_g \psi_g$  vs. time (in sec)

Note, that this experimental waveform fades out around  $t=80 \mu\text{s}$  after the breakdown, close to the time that CT formation is complete.

The helicity vs time obtained by integrating Eq.(4) is shown in Fig.4 for two different time constants.

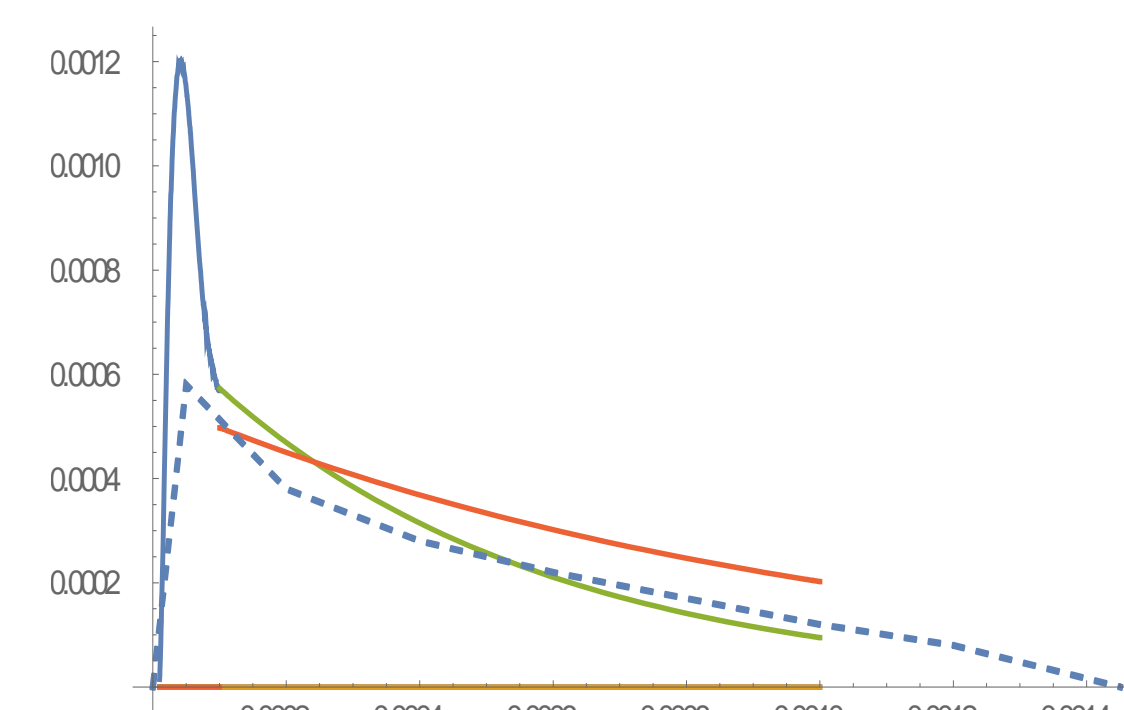


Fig 4. Helicity vs. time: The green and red curves have decay times of 0.5 and 1 msec respectively. The dashed line is the helicity obtained from a CORSICA reconstruction of the equilibrium based on surface Mirnov probes measurements from experiment.

It can be seen that the peak helicity content estimated by the CORSICA equilibrium code is about only a half of the electrostatic input of helicity calculated from the voltage and poloidal flux applied between the electrodes.

## Analyses Of "Bubble-out" Threshold

Another interesting result that can be deduced by assuming a force-free Taylor state is the stretching current  $I_{st}$  threshold, the current required at the gun electrodes to push the poloidal flux into the flux conserving area. Assuming no external toroidal field, the relation obtained in [6] is:

$$I_{st} = \frac{8 \psi_{gun}^2}{\mu_0^2 d^2 I_{tf}} \quad (5)$$

where  $d$  is the distance between the inner and outer electrodes of the coaxial gun and  $I_{tf}$  is the current through the central rod. This formula is plotted with 3 values of the poloidal flux and  $d=0.065 \text{ m}$  in Fig.5a.

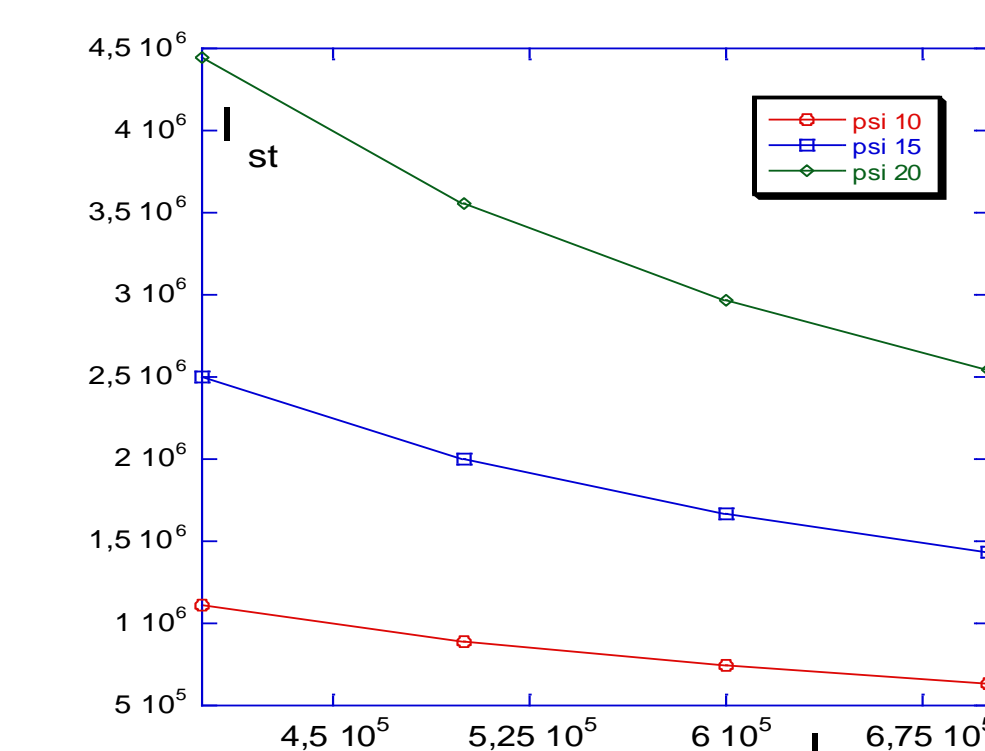


Fig 5a.  $I_{st}$  vs.  $I_{tf}$  for 10, 15 and 20 mWb of poloidal flux.

A formula that relates the shaft current  $I_{tf}$  and the gun current  $I_{gun}$  with different poloidal fluxes is presented in [7] without much explanation:

$$I_{st} = \frac{\lambda_{gun} \psi}{\mu_0} \quad (6)$$

where  $\lambda_{gun} = (\lambda_{co}^2 + \lambda_R^2)^{1/2} - \lambda_R$   
 $\lambda_{co} \sim I_{eig}$ , and  $\lambda_R = \frac{\mu_0}{\psi_{gun}} I_{tf}$

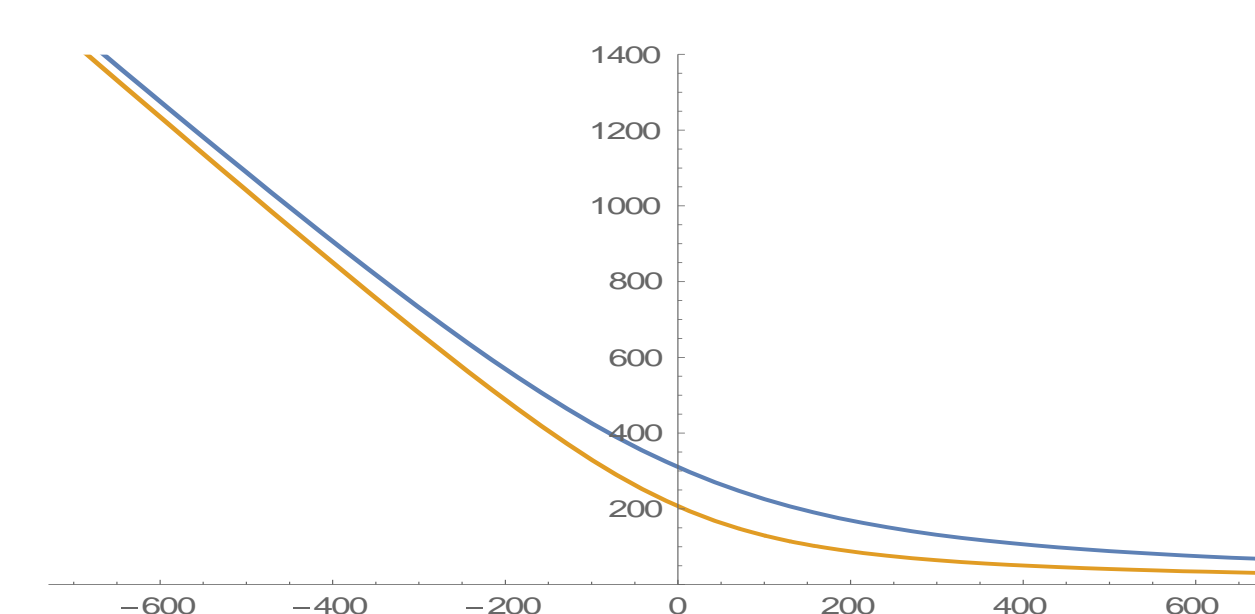


Fig 5b.  $I_{gun}$  vs  $I_{tf}$  for 15 mWb (blue line) and 10 mWb (orange line) poloidal flux. The currents are in kA.

Depending on the mutual directions of  $I_{gun}$  and  $I_{tf}$ , the toroidal magnetic field can "help" or hamper the bubble out. To push the same amount of flux, a shaft current with the same sign as the gun current is desirable because it reduces the required gun current considerably.

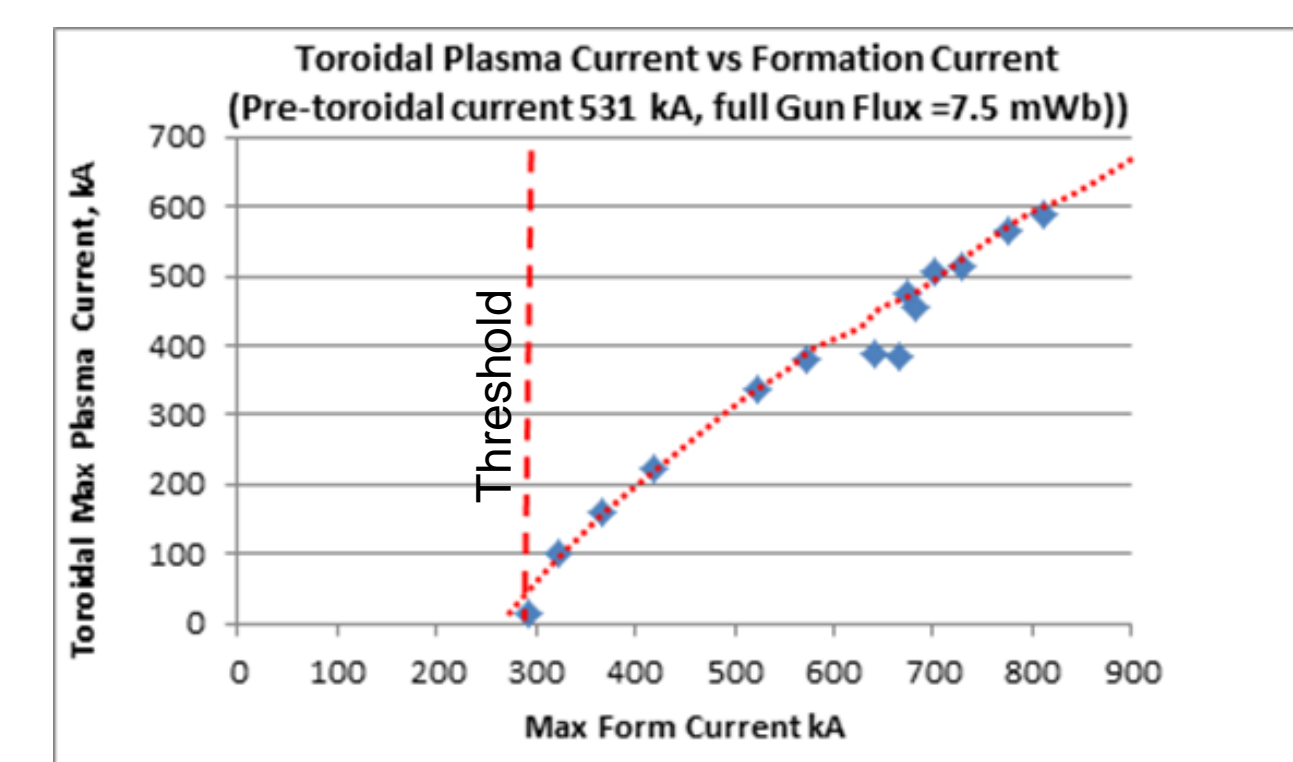


Fig 6. The 250 kA threshold for "bubble-out" determined by experiment in a smaller SPECTOR-type machine is consistent with the prediction of 270 kA by the formula (5). However, it is greater than the theoretically-predicted threshold of 114 kA by formula (6).

## Helicity Balance And Plasma Resistive Dissipation

The helicity dissipation and the poloidal energy dissipation time scales can also be estimated from the Corsica simulations as:

$$\text{helicity dissipation rate } d_K = \frac{1}{K} \frac{dK}{dt} [\mu\text{s}^{-1}] ,$$

$$\text{poloidal energy dissipation rate } d_W = \frac{1}{W_{pol}} \frac{\Delta W_{pol}}{\Delta t} [\mu\text{s}^{-1}] .$$

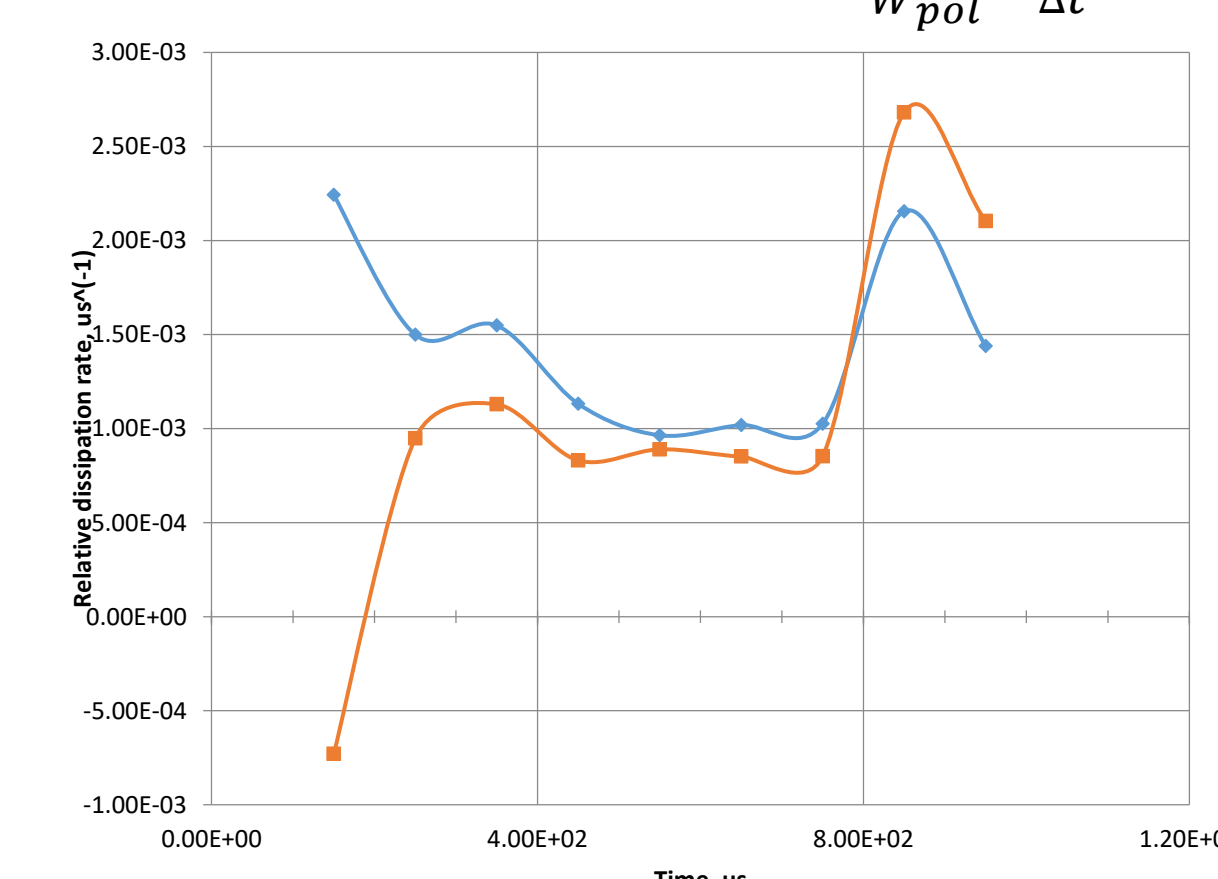


Fig 7.  $d_K$  (blue diamonds) and  $d_W$  (brown squares) vs. time as deduced from the CORSICA simulations.

The fact that the two time scales are similar is equivalent to the statement that the dynamo term is very small in this system. Therefore, there is likely no significant flux amplification effect. This is a reasonable result, since it is known that the dynamo is not strong in a tokamak-like plasma, unlike spheromaks and reverse field pinches.

It should be noted that an IRE (Internal Relaxation Event) occurred around  $t=0.8 \text{ ms}$  ( Fig 7). During an IRE, the energy dissipation rate somewhat exceeds the dissipation of helicity, so associated dynamo activity in the plasma is possible.

The helicity dissipation deduced from Fig. 7 is in the range 0.3-0.6  $\text{s}^{-1}$  (depending on time). On the other hand, when the formation of a CT is finished, the helicity dissipation can be estimated from the Ohmic parallel current term (see Eq.(2) and Eq.(3)).

$$\frac{dK}{dt} = 2 \int_{core-sheath} \eta_{||} \mathbf{J} \cdot \mathbf{B} d^3r$$

By assuming a simple constant  $J/B$  model [9] and the Spitzer resistivity given by:

$\eta_{||} = Z_{eff} * 0.5 * 10^{-3} * T_0^{-1.5} * (1 - r^a)^{-1.5} [\text{W m}]$   
 with  $T_0$  being the on-axis temperature in eV, and  $Z_{eff}$ , the effective dissipation integral over the volume can be calculated. The results are plotted in Fig. 8 as a function of  $T_0$ , for  $Z_{eff} = 2,3$  and  $a = 2,4$ .

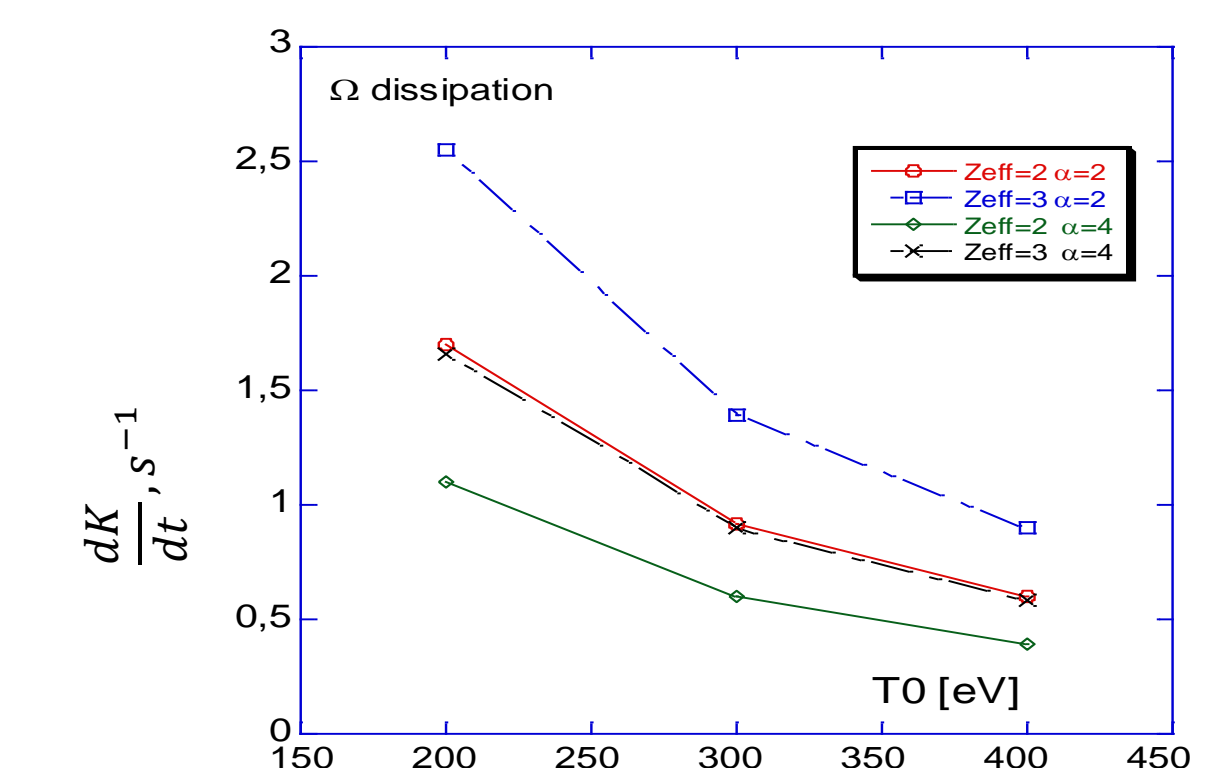


Fig 8. Volume integral of the ohmic helicity dissipation vs  $T_0$ .

The helicity dissipation deduced from Fig 7 ( $0.4 - 0.6 \text{ s}^{-1}$ ) is of the same order as the helicity dissipation deduced from ohmic dissipation (Fig 8). Since the values of  $Z_{eff}$  (2-3) and  $T_0$  (400-450 eV) are within the range measured in SPECTOR, Ohmic dissipation may account fully for the measured helicity dissipation without the need to invoke anomalous dissipation mechanisms or enhanced losses to the surrounding walls.

## Discussion And Summary

We found that the helicity injected in the system is about 50% of the available electrostatic helicity.

The helicity and energy balance exhibit the same behavior, implying that flux amplification or a dynamo mechanism is small or absent (at least for times when the equilibrium reconstruction is reliable). High-q tokamaks do not seem to be prone to these effects.

We found that Ohmic dissipation is enough to account for the helicity losses without the need to invoke an anomalous mechanism.

We have begun to compare the experiment to different models for the amount of bubbled-out flux produced by a given gun current. More experiments oriented to this end are needed.

We found that during IRE the helicity is better conserved than energy (Fig.7), which, if confirmed, is an important fact from a basic physics point of view.

## References

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