MHD Stability of a Magnetized Target During Non-Self-Similar Compression

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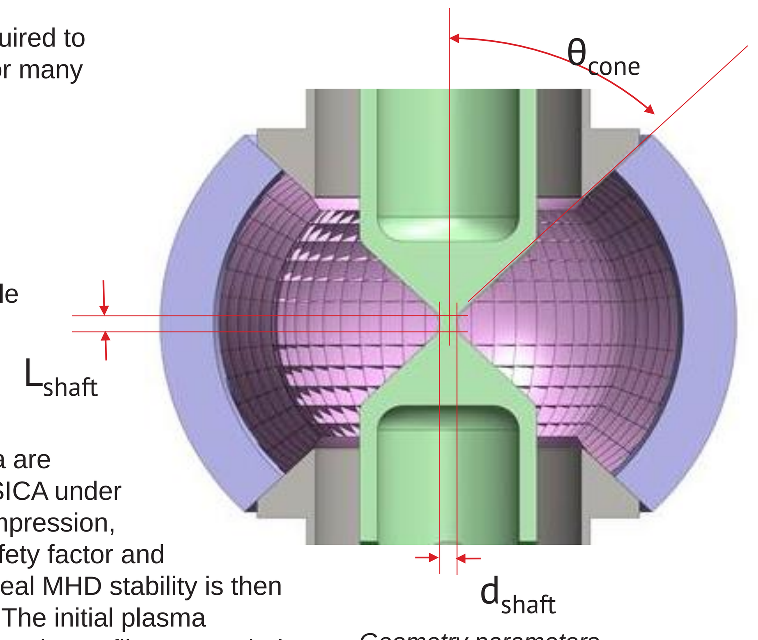
Introduction

General Fusion is designing a magnetized target fusion prototype that will compress a toroidal plasma inside a liquid metal cavity to heat it to fusion conditions [1]. A magnetized plasma will heat adiabatically if rapidly compressed in a flux conserver (FC) following simple scalings in self-similar geometry.

In practice, variation of geometry from self-similarity complicates the equilibrium and stability calculations that drive the design process. Here we present computations of the plasma stability evaluated for the entire compression of realistic FC geometries.

Stability analysis is required to inform design values for many parameters.

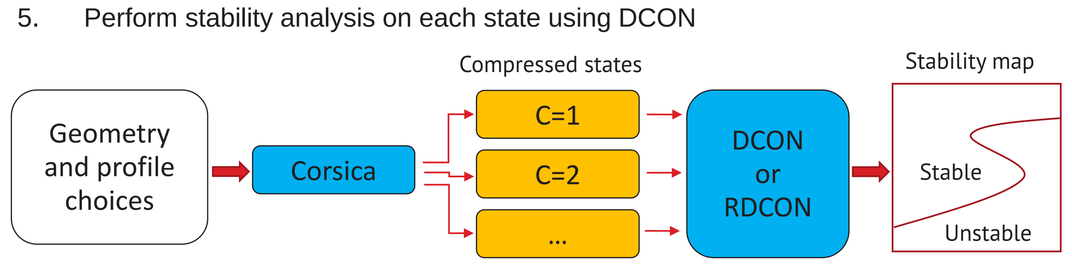
- Shaft diameter
- Shaft length
- Cone angle
- Current density profile
- Pressure profile
- Shaft current



Geometry parameters

Sequences of equilibria are generated using CORSICA under ideal and adiabatic compression, conserving both the safety factor and entropy profiles. The ideal MHD stability is then computed with DCON. The initial plasma pressure and current density profiles are varied to optimize the stability boundaries.

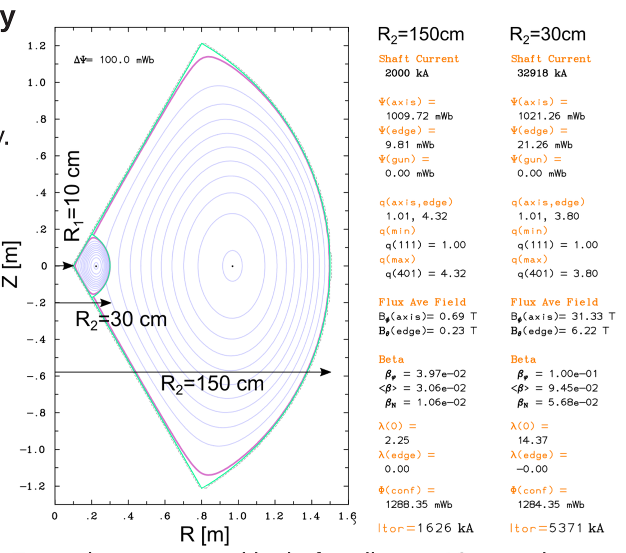
1. Choose a flux-conserver geometry
2. Choose initial pressure profile and plasma current profile
3. Solve for plasma equilibrium using CORSICA
4. Solve for compressed states using ideal MHD approximation
5. Perform stability analysis on each state using DCON



- Shaping affects both the stability and transport. But here we focus on shaping effects on stability during compression.
- Main controls during experimental operation are initial profiles (by relaxation rate and wait time) and shaft current during compression.
- Profiles were expected to be L-mode like with a peaked current density and pressure. This assumption was found to be incorrect, as our best case has a non-monotonic current density (reversed lambda near the wall). We have some control over the pressure peaking and the shape of the lambda profile.

Example compression geometry

- Compression is not exactly self similar. Shaping of boundary during compression handled parametrically.
- The effect of shaft length and diameter is more significant in the compressed state.
- A longer shaft improves plasma elongation and triangularity in compressed state, but reduces maximum volume compression.



Example geometry with shaft radius r_1=10cm, sphere radius r_2=1.5m, and shaft length l_{sh}=20cm.

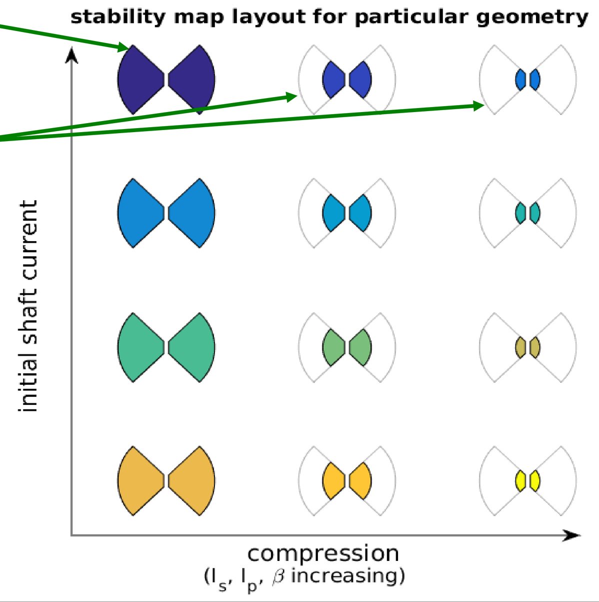
- In leftmost column are initial equilibria varying by shaft current. The plasma pressure and current profiles are all the same.
- As we move rightward the equilibrium is being compressed. Currents and β are increasing.
- Flux is frozen in plasma, so safety factor (q) is conserved.

$$\frac{d\Phi}{d\psi} = q(\bar{\psi}) = \text{const}$$

- Adiabatic compression is assumed, so adiabatic constant is conserved on flux surfaces.

$$P(\bar{\psi})\left(\frac{dV(\bar{\psi})}{d\bar{\psi}}\right)^{\gamma} = \text{const}$$

stability map layout for particular geometry



Equilibrium Compression with CORSICA

CORSICA [2] is an extensible software system for simulating toroidal magnetic fusion devices. We use it to calculate plasma equilibria by evaluating the Grad-Shafranov equation subject to specified boundary conditions.

We model the wall as a superconducting flux conserver that excludes all magnetic fields.

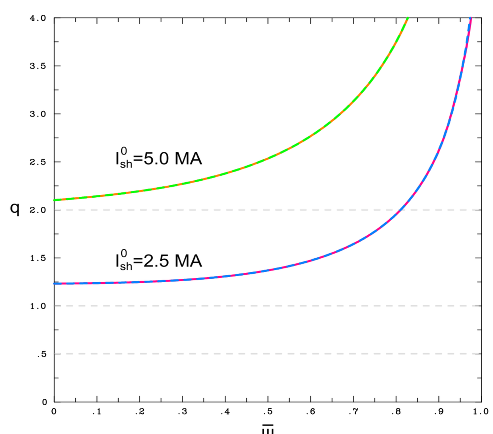
Compression must be faster than magnetic decay and heat loss, so we conserve the plasma safety factor, q(ψ), and adiabatic constant, pV^{γ}, profiles.

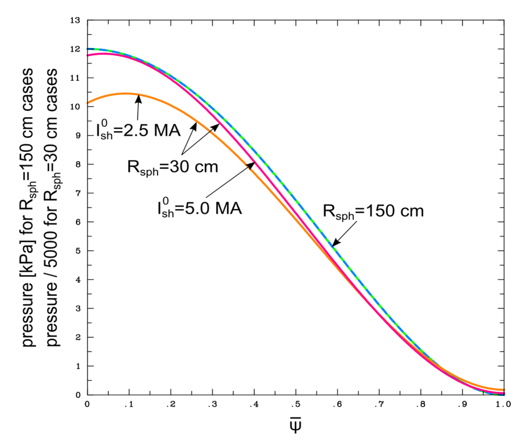
During ideal compression plasma pressure increases faster than magnetic pressure so β increases. This can result in pressure-driven instability. To make pressure-driven instabilities easily visible, we assume a high starting temperature, 700 eV - 1 keV.

Temperature is assumed to increase adiabatically according to volume compression ratio.

Plasma Properties During Compression

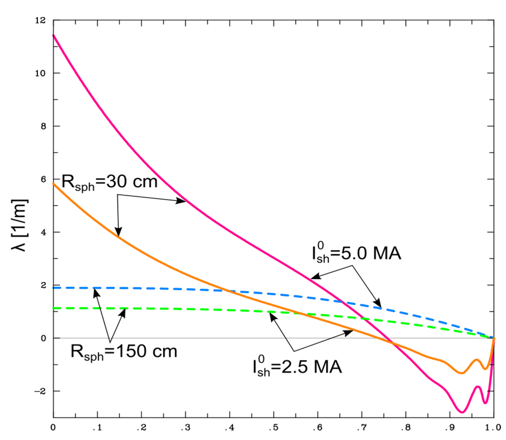
- On the right is the q profile for two starting shaft currents and two different compression ratios.
  Uncompressed: R_0/R=1 (blue/green)
  Highly compressed: R_0/R=5 (orange/red)
- The q profile is conserved in this analysis, so it does not change during compression.





- On the left is the pressure profile for two starting shaft currents. Each case starts with the same profile, but flux surfaces compress differently, so final profiles are different. Adiabatic compression goes as P^5, but most compression is near the core, so peak pressure increases faster. (5000 > 5^5 = 3125).

- On the right, we show the lambda=$J_{||}$/|B| profiles before and after compression. They start peaked (green/blue), but they grow in magnitude and develop a negative value near the edge. The negative value is maintaining a lower q value near the wall because the toroidal field is being compressed faster than the poloidal field.

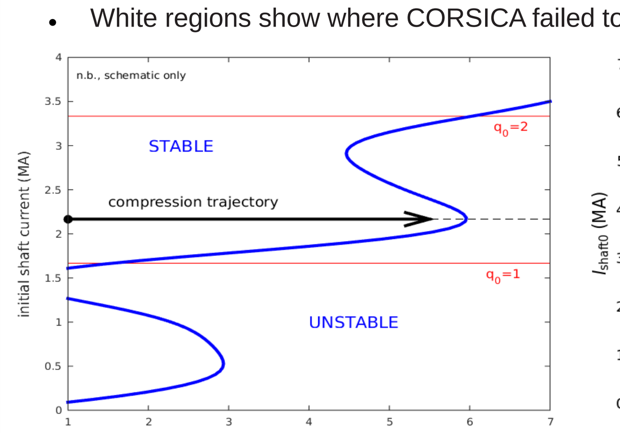


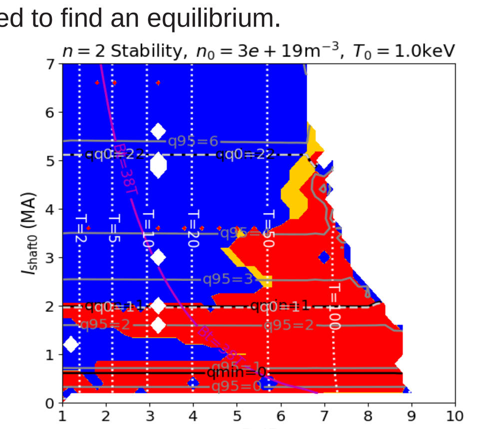
Stability Analysis with DCON

- DCON [3] calculates the ideal MHD stability of an axisymmetric toroidal plasma by using a generalization of Newcomb's criterion for cylindrical plasmas. For each toroidal mode number n, the potential energy δW is minimized and then poles in a critical determinant indicate the presence of ideal instabilities. Both internal (fixed-boundary) and external (free-boundary) interchange modes can be detected with this method.
- RDCON [4] is a new version of DCON that evaluates resistive stability by extending the zero-pressure analysis of Johnson and Greene. It checks for the presence of resistive interchange and tearing modes and returns their growth rates given a resistivity profile.
- The stability boundary depends on geometry and choices for starting lambda and pressure profiles.

Stability map schematic

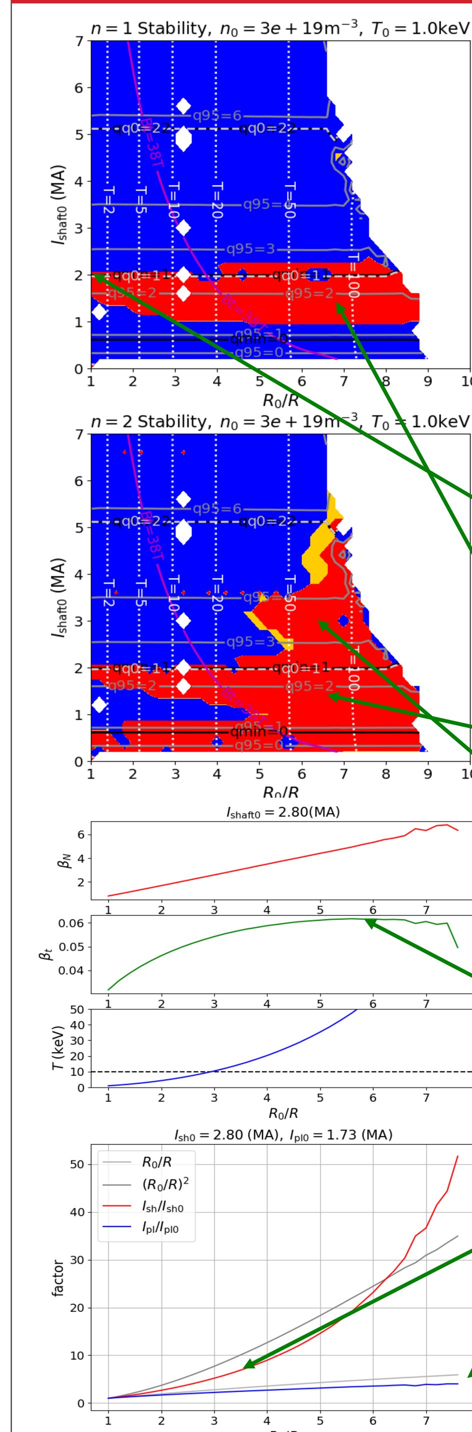
- Blue is ideal MHD stable (δW>0). Orange is free-boundary unstable (δW<0).
- Red is fixed-boundary unstable.
- Vertical dotted white lines are temperature contours (keV).
- Horizontal gray lines are contours of q profile
- Magenta line is maximum B-field limit at the shaft. At 38 Tesla, liquid lithium begins to heat rapidly and lose conductivity. The shaft is carrying at least 20MA of shaft current and 500MPa of pressure.
- White regions show where CORSICA failed to find an equilibrium.

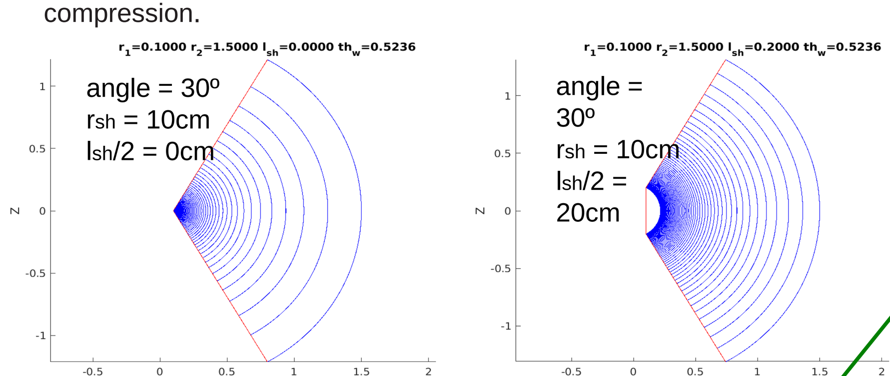


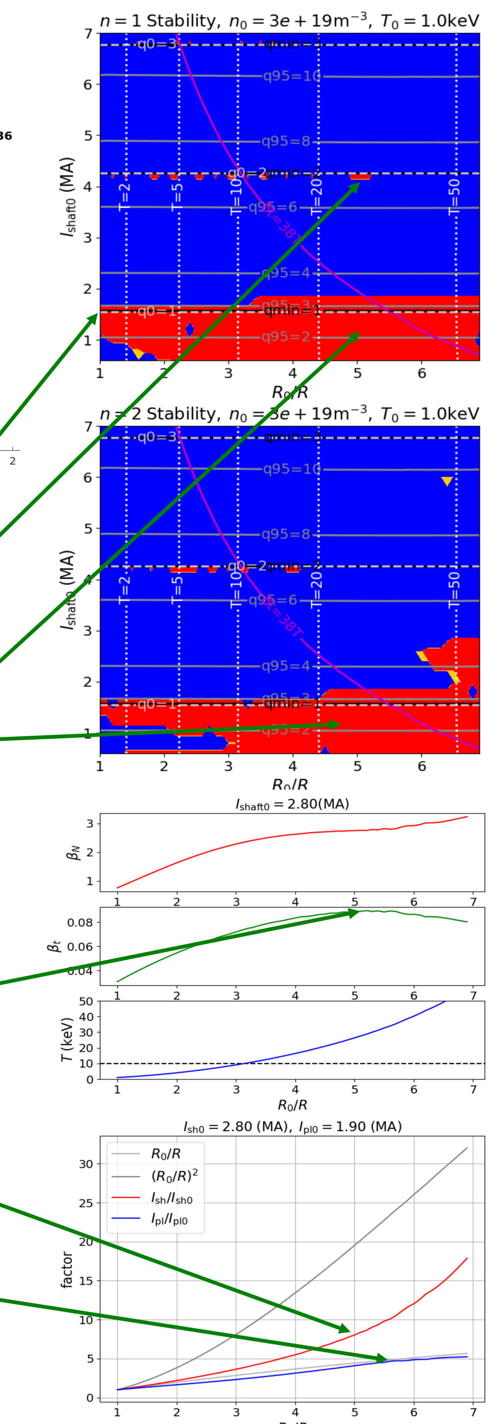


Geometry Effects

Here we look at the effects of changing the length of the straight shaft section, which increases elongation over time and limits the maximum compression.





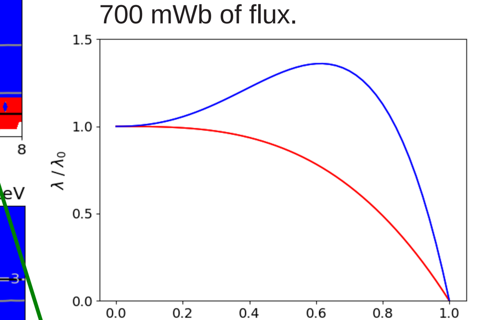


- Geometry affects the stability primarily by altering the q profile. The longer shaft reduces the q profile for a given initial shaft current.
- There is some n=1 instability seen when the q0=2.
- There is a robust region of n=1 and n=2 instability when 0.6<q0<1.
- There is n=2 instability when q_0<2 at high compression (R_0/R>6).
- The assumption of adiabatic compression means that the temperature increases rapidly with compression.
- Beta toroidal reaches a maximum at R0/R~5.
- Shaft current grows faster than R0/R to maintain the q profile near the wall. It exceeds 20MA at about R0/R=3.5 on the left and R0/R=5 on the right, which causes the field to exceed 38 Tesla. In reality, this shaft current would cancel with the negative lambda near the wall.
- Plasma current grows nearly proportional to R0/R.

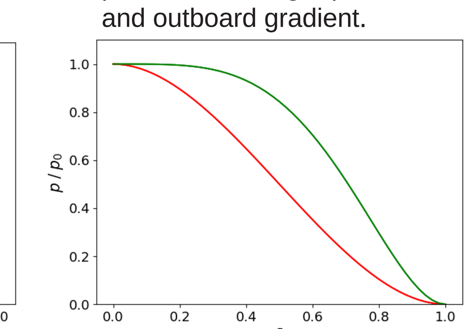
Current Density and Pressure Profile Effects

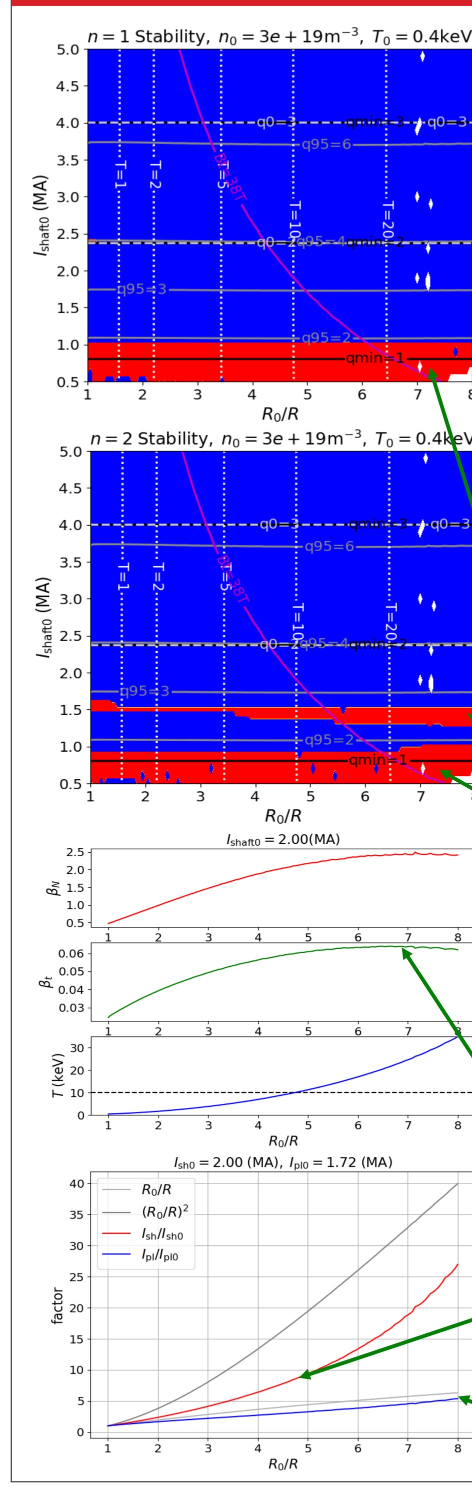
Here we present the effects due to modifying the lambda and pressure profiles.

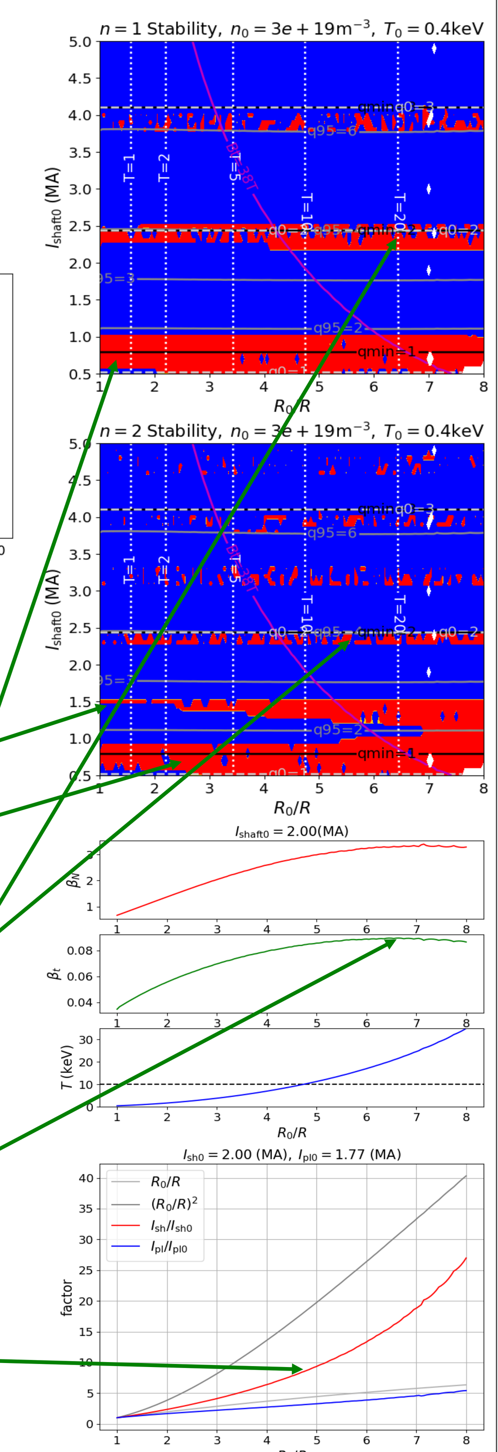
The cases above use the red peaked lambda profile with 1mWb of poloidal flux. These cases use the blue hollow lambda profile with 700 mWb of flux.



The left case uses the red pressure profile and the right case uses the green pressure profile with a larger plateau and outboard gradient.







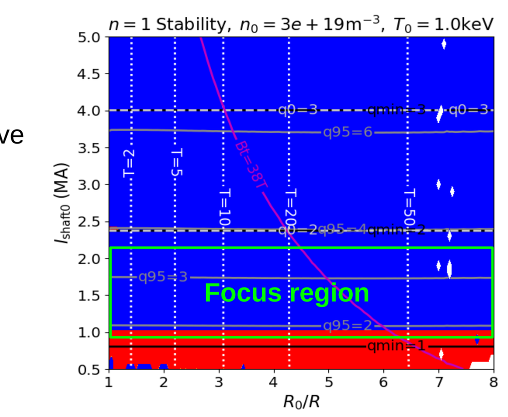
- As with modifying the geometry, stability is affected by the changes made to the q profile. The hollow lambda profile reduces the q-profile for a given initial shaft current, though it is a subtle change.
- There is a new band of n=2 instability at q0=3/2.
- There is still a robust region of n=1 and n=2 instability when q0<1.
- When the pressure gradient is increased towards the outer wall, n=1 instabilities appear when integer q values enter the plasma from the wall, and n=2 instabilities appear with q/2=integer values enter.
- In these cases, the plasma starts at 0.4 keV, and since the temperature increases at the same rate, the final temperature and beta is lower.
- Beta toroidal reaches a maximum at about R0/R=6.5.
- Shaft current grows faster than R0/R to maintain the q profile near the wall. It exceeds 20MA at about R0/R=5.0, which causes the field to exceed 38 Tesla. In reality, this shaft current would cancel with the negative lambda near the wall.
- Plasma current grows nearly proportional to R0/R.

Resistive Stability

Even when ideal stable, can be resistively unstable to tearing modes.

Application of Resistive DCON code for resistive stability analysis of n=1.

Focus on the region of experimental interest, ideal stable with viable shaft current.



Pure Interchange Parity

$$\mathbf{D}' \equiv \frac{1}{2}\begin{bmatrix} A' & B' \\ \Gamma' & \Delta' \end{bmatrix}$$

Coupling

Pure Tearing Parity

$$\mathbf{D}(\gamma) \equiv \frac{1}{2}\begin{bmatrix} A(\gamma) & 0 \\ 0 & \Delta(\gamma) \end{bmatrix}$$

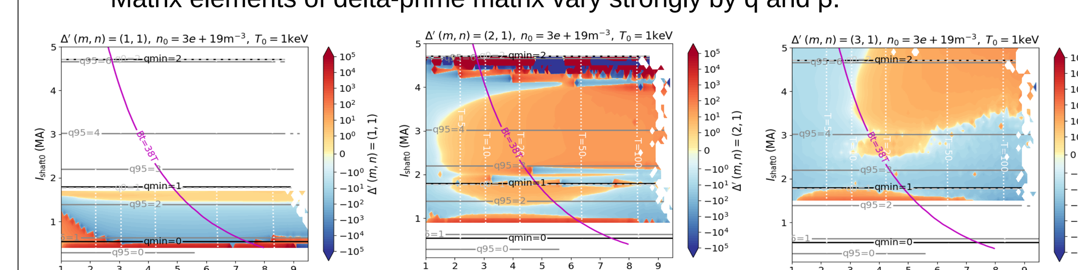
Numerical inner layer solution computed at each surface

Solve determinant for γ.

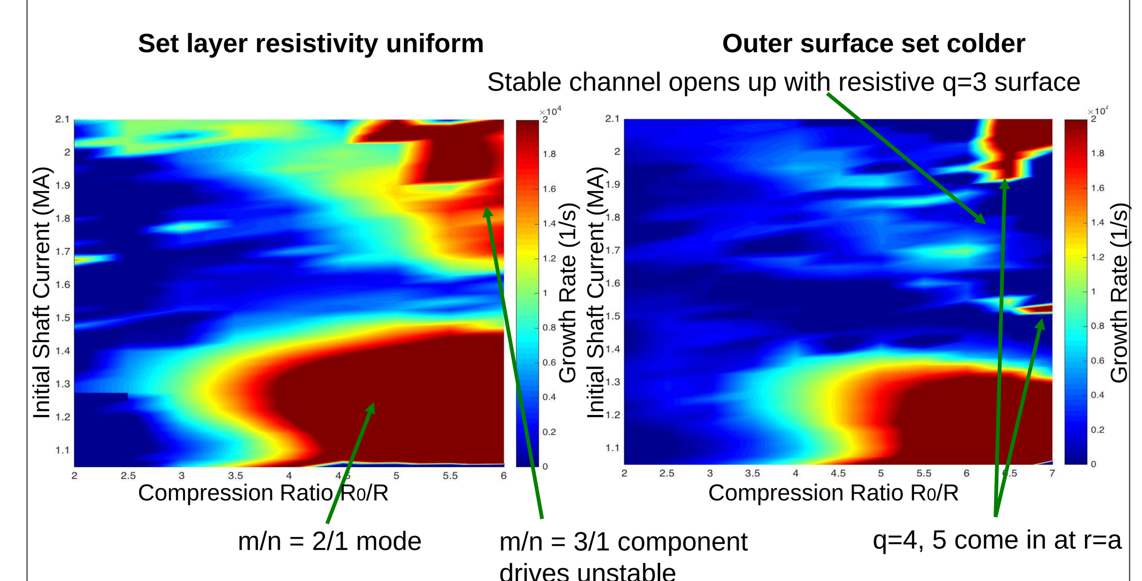
$$det[\mathbf{D}' - \mathbf{D}(\gamma)] = 0$$

Numerically solves resistive layer equations in full toroidal geometry for growth rate γ

Matrix elements of delta-prime matrix vary strongly by q and β.



- Layer resistivity set as a function of T -> higher beta leads to higher S at lower R.
- Outer surface resistivity also important. Lower T on outer surface opens up stable region to low R.



Conclusions of resistive stability analysis

We have used RDCON to evaluate a region that is n=1 ideal MHD stable. In this map of compression trajectories, all but a narrow range of starting shaft currents exhibited resistive n=1 instabilities with growth rates much faster than the speed of compression. However, by cooling the edge, a much wider range of conditions were found that permit a plasma to be compressed fully without encountering n=1 resistive instabilities.

Summary

By assuming an adiabatic process and conserving the q profile with CORSICA, we have developed a robust method to approximate plasma properties under rapid compression. This allows us to explore plasma geometry and profile parameter space to find a stable compression method.

Geometry was found to affect the stability mainly via the q profile. Most cases with q_0<1 are found to be n=1 unstable, while those with q_0>1 can be susceptible to low n modes.

Lambda and pressure profiles have a greater affect on the structure of the stability map. As expected, greater pressure gradients can destabilize the plasma even in high shaft current/high q operating regimes. However, viable initial equilibria are found that remain stable throughout the compression.

We expected beta to increase proportional to the compression ratio, but were surprised to find that beta reaches a limit when the compression ratio is around 5, so beta limits should not be a serious problem at high compression.

Plasma stability tends to increase by adding more shaft current. However, shaft current requires expensive capacitor banks. We have found that a lower shaft current can be accommodated by using a hollow lambda profile, small shaft radius, and longer shaft.

Even considering resistive stability, we find conditions that permit a plasma to be compressed fully without encountering any n=1 or n=2 instabilities.

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[2] J.A. Crotinger, et al., LLNL Report UCRL-ID-126284 (1997).
[3] A.H. Glasser, J.M. Greene, and J.L. Johnson, Phys. Fluids 18 (1975) pp 875.
[4] A.H. Glasser, Z.R. Wang, and J.-K. Park, Phys. Plasmas 23, 112506 (2016).