One-Dimensional MHD Simulations of MTF Systems with Compact Toroid Targets and Spherical Liners

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INTRODUCTION

- One-dimensional (1D) MHD code is developed in General Fusion (GF) for coupled plasma-liner simulations in magnetized target fusion (MTF) systems.
- The main goal of the code is to provide a simple tool for searching optimal parameters of MTF reactor, in which spherical liquid metal liner compresses compact toroid (CT) plasma.
- The code uses Lagrangian specification for both liner and plasma with self-consistent description of poloidal and toroidal magnetic fields. Different transport and liner motion models are implemented, this allows for comparison with ongoing GF experiments.
- We performed a series of parameter scans in order to establish the underlying dependencies of MTF system and find the optimal GF reactor prototype design point.

MODEL

Geometry

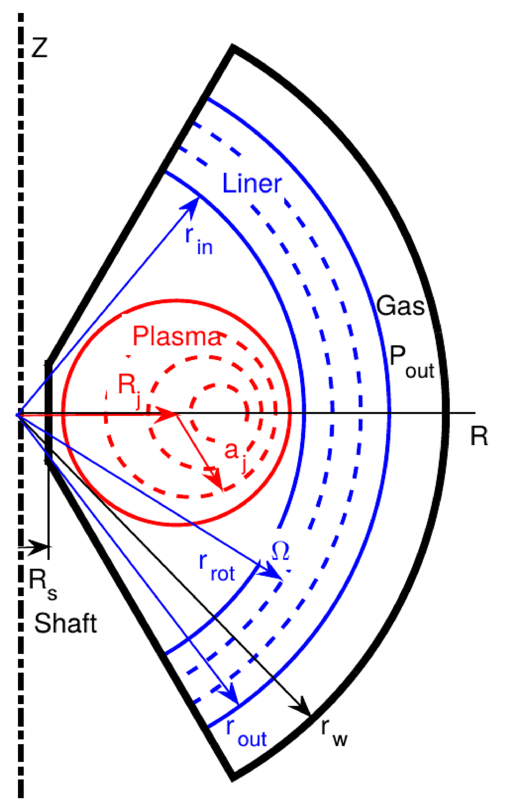

Figure 1: Model geometry.

- The system is axisymmetric around central Z-axis and has up-down symmetry with respect to equator.
- Solid metal central shaft along Z-axis is a vertical cylinder of radius R_s.
- Plasma is discretized as a set of nested tori with circular cross sections and fixed number of particles between them. The j-th torus has major radius R_j and minor radius a_j.
- Liquid metal liner is discretized as a set of spherical shells with fixed masses. The *j*-th shell is labeled by spherical radius r_j. Part of the liner between r_{in} and r_{rot} is rotating toroidally with angular velocity Ω *(r,t)*.
- Coordinates R_j, a_j and r_j are Lagrangian, they are fixed to given fluid parcels but can change in time. All physical quantities are functions of time and these Lagrangian coordinates.
- Driving gas at pressure P_{out} pushes the liner, thus compressing plasma.

Plasma Equations

Axisymmetric magnetic field in plasma:

$$\mathbf{B} = \nabla\psi(a) \times \nabla\phi + F(a)\nabla\phi$$

Assuming fast (Alfvenic time-scale) equilibration, plasma satisfies the Grad-Shafranov equation at every moment of compression:

$$\left\langle \Delta^*\psi\nabla\psi + F\nabla F + \mu_0 R^2\nabla(p_e+p_i)\right\rangle_\psi = 0$$

Dynamical equations are written in terms of passive scalars – quantities that are transferred passively with plasma shells and do not change in the absence of diffusion and sources/sinks of energy. They are: poloidal flux ψ, toroidal flux Φ, electron and ion entropies, s_e and s_i. Angled brackets denote appropriate averaging along flux surface.

$$\begin{aligned}
\frac{d\psi}{dt} &= -\mu_0\eta\langle R j_{tor}\rangle_\psi \\
\frac{d\Phi}{dt} &= -2\pi\mu_0\eta\langle a j_{pol}\rangle_\psi \\
\frac{ds_e}{dt} &= \frac{2}{3n^{5/3}}\left\langle \nabla\cdot(\kappa_e\nabla T_e) + \mu_0\eta\left(j_{pol}^2 + j_{tor}^2\right) - \frac{3m_e}{m_i}\frac{Zn(T_e-T_i)}{\tau_{ei}} + Q_{extra}\right\rangle_\psi \\
\frac{ds_i}{dt} &= \frac{2}{3n^{5/3}}\left\langle \nabla\cdot(\kappa_i\nabla T_i) + \frac{3m_e}{m_i}\frac{Zn(T_e-T_i)}{\tau_{ei}}\right\rangle_\psi
\end{aligned}$$

Components of current density:

$$j_{tor} = -\frac{1}{\mu_0}\frac{\Delta^*\psi}{R}, \quad j_{pol} = -\frac{1}{\mu_0}\frac{F'_a}{R}$$

Definition of toroidal flux F and inductance of toroidal shell L_j:

$$F = \frac{d\Phi}{dL}, \qquad L_j = \int_0^{a_j}\int_0^{2\pi}\frac{a\,d\theta\,da}{R_j + a\cos\theta} = 2\pi\left(R_j - \sqrt{R_j^2 - a_j^2}\right)$$

Density and volume of *j*-th toroidal plasma shell (total number of particles dN_j within each shell is conserved during dynamics):

$$n_j = \frac{dN_j}{dV_j}, \qquad dV_j = 2\pi^2\left(a_j^2 R_j - a_{j-1}^2 R_{j-1}\right)$$

Thermodynamical quantities:

$$p_e = s_e n^{5/3}, \quad p_i = s_i n^{5/3}$$

$$T_e = \frac{p_e}{Zn}, \qquad T_i = \frac{p_i}{n}$$

The last term Q_{extra} in the electron entropy equation denotes other possible sorces and sinks of energy, such as heating due to collisions with fusion alpha-particles and radiative losses due to Bremsstrahlung.

Plasma Transport Coefficients

Transverse Spitzer resistivity (magnetic diffusivity): $\eta\left[\frac{\mathrm{m}^2}{\mathrm{s}}\right] = 820 Z\ln\Lambda(T_e[\mathrm{eV}])^{-3/2}$

Thermal conductivities:

$$\begin{aligned}
\kappa_e\left[\frac{\mathrm{J}}{\mathrm{eV\,m\,s}}\right] &= Z\frac{3}{2}1.6\times10^{-19}n_0[\mathrm{m}^{-3}]\chi_e\left[\frac{\mathrm{m}^2}{\mathrm{s}}\right] \\
\kappa_i\left[\frac{\mathrm{J}}{\mathrm{eV\,m\,s}}\right] &= \frac{3}{2}1.6\times10^{-19}n_0[\mathrm{m}^{-3}]\chi_i\left[\frac{\mathrm{m}^2}{\mathrm{s}}\right]
\end{aligned}$$

Here n_0 is the initial peak ion density, χ_e and χ_i are the electron and ion thermal diffusivities. All these quantities in present simulations are assumed to be constant, although other choices are available (such as classical Braginskii transport). Electron-ion collisional time (for thermal equilibration)

$$\tau_{ei}[\mathrm{s}] = \frac{3.44\times10^{11}(T_e[\mathrm{eV}])^{3/2}}{Z^2\ln\Lambda\, n[\mathrm{m}^{-3}]}$$

Liner Equations

Axisymmetric magnetic field in liner (*r* – spherical radius, θ – polar angle):

$$\mathbf{B} = \nabla\psi(r,\theta)\times\nabla\phi + F(r,\theta)\nabla\phi$$

Equation of radial motion (here *R*=*r* sinθ – cylindrical radius):

$$\rho\frac{dv_r}{dt} = \rho\left\langle\frac{v_{tor}^2}{r}\right\rangle_\theta - p'_r - \left\langle\frac{\Delta^*\psi\,\psi'_r}{R^2} + \frac{FF'_r}{R^2}\right\rangle_\theta + \left(\frac{\nu\rho}{r^2}(r^2 v_r)'_r\right)'_r$$

Dynamical equations for passive scalars – angular momentum, poloidal flux ψ, toroidal flux Φ and entropy *s*. Angled brackets denote appropriate averaging in polar angle θ (along spherical surface).

$$\begin{aligned}
\frac{d}{dt}\langle R v_{tor}\rangle_\theta &= 0 \\
\frac{d\psi}{dt} &= -\mu_0\eta\langle R j_{tor}\rangle_\theta \\
\frac{d\Phi}{dt} &= -\pi\mu_0\eta\langle r j_{pol}\rangle_\theta \\
\frac{ds}{dt} &= \frac{(\gamma-1)}{\rho^\gamma}\left\langle\nabla\cdot(\kappa\nabla T) + \mu_0\eta\left(j_{pol}^2 + j_{tor}^2\right) + \nu\rho\left(\frac{(r^2 v_r)'_r}{r^2}\right)^2\right\rangle_\theta
\end{aligned}$$

Components of current density: $j_{tor} = -\frac{1}{\mu_0}\frac{\Delta^*\psi}{R}, \quad j_{pol} = -\frac{1}{\mu_0}\frac{F'_r}{R}$

Definition of toroidal flux Φ in liner: $F = \frac{1}{\pi}\frac{d\Phi}{dr}$

Density and volume of *j*-th spherical shell (total mass within each shell is conserved):

$$\rho_j = \frac{dM_j}{dV_j}, \qquad dV_j = \frac{4\pi}{3}\left(r_j^3 - r_{j-1}^3\right)$$

In present simulations thermal conductivity κ and magnetic diffusivity (resistivity) η of liner are assumed to be constant, although other choices are available in the code based on look-up tables for specific temperature and pressure intervals.

Two models are available for the liquid metal equation of state: compressible and incompressible.

Compressible model uses Mie-Grueneisen equation of state[1] with "cold" (elastic) p_c and thermal p_T components of pressure:

$$p_c = \frac{\rho_0 c_0^2}{k}\left(\left(\frac{\rho}{\rho_0}\right)^k - 1\right), \quad p_T = s\rho^\gamma$$

$$p = p_c + p_T, \quad T = T_0 + \frac{p_T}{(\gamma-1)\rho c_V}$$

where ρ_0 – initial density, T_0 – initial temperature, c_0 – initial speed of sound, *k* – hardening power, γ – specific heat ratio, c_V – heat capacity at constant volume.

Table 1: Properties of liquid metals proposed for liner

Liquid metal	T_0^{melt}, K	ρ_0, kg/m^3	c_0, km/s	c_V, J/kg·K	κ, W/m·K	η, m/s^2	γ	k
Lithium	453.6	516	4.5	4169	44.0	0.20	1.85	1
Lead	600.6	10678	2.0	148	14.8	0.76	3.77	3

Incompressible model assumes constant density and divergence-free velocity (dot denotes time derivative):

$$\rho = \rho_0, \qquad v_r = \frac{\dot r_{in} r_{in}^2}{r^2}$$

This leads to simplified equation of liner radial motion (integrated from r_{in} to r_{out}):

$$\left(\ddot r_{in} r_{in} + 2\dot r_{in}^2\right)\left(1 - \frac{r_{in}}{r_{out}}\right) - \frac{\dot r_{in}^2}{2}\left(1 - \frac{r_{in}^4}{r_{out}^4}\right) = \int_{r_{in}}^{r_{out}}\left\langle\frac{v_{tor}^2}{r}\right\rangle_\theta dr - \frac{\Delta P}{\rho_0}$$

$$\Delta P = P_{out} + \int_{r_{in}}^{r_{out}}\left\langle\frac{\Delta^*\psi\,\psi'_r}{R^2} + \frac{FF'_r}{R^2}\right\rangle_\theta dr, \qquad r_{out}^3(t) - r_{in}^3(t) = r_{out}^3(0) - r_{in}^3(0)$$

Liner motion is determined by combined effect of centrifugal force, driving gas pressure P_{out} and magnetic pressure drop between inner and outer liner surfaces.

NON-COMPRESSION DYNAMICS

- As a validation test we applied the developed code to simulate non-compression dynamics in SPECTOR1 – one of General Fusion's plasma experiments.
- SPECTOR1 is a spherical tokamak with plasma formed by Marshall gun discharge into pre-existing toroidal field. Flux conserver has spherical shape with radius r_w=19 cm, and center shaft electrode is cylindrical with R_s=1.3 cm.
- A typical SPECTOR1 shot with its simulation is shown on Fig. 2. As initial input for simulation we used the following parameters: uniform density $n_0=10^{14}$ cm^{-3}, total poloidal flux in plasma ψ_0=26.5 mWb, shaft current I_s=0.765 MA. To match the experimental poloidal magnetic field decay we chose electron thermal diffusivity of χ_e=250 m^2/s and increased Spitzer resistivity by a factor of 3.5.

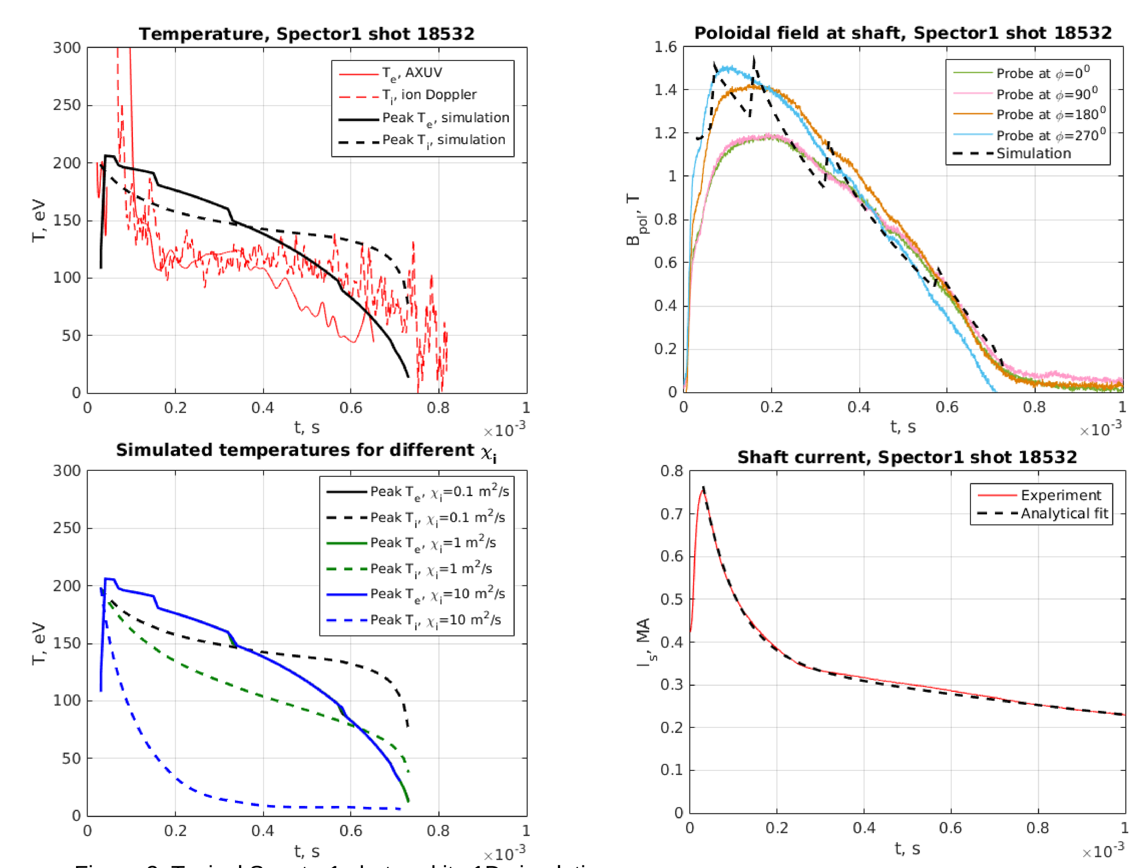

Figure 2: Typical Spector1 shot and its 1D simulation.

Change of ion thermal diffusivity χ_i changes only ion temperature dynamics, but neither electron temperature nor magnetic field are affected much. From our results it follows that in order for ions to have temperature plateau at T_i~100 eV for ~500 μs (as observed in experiment) they must have thermal diffusivity of χ_i<10 m^2/s.

PROTOTYPE PARAMETERS SCANS

GF Prototype program is aimed at achieving peak ion temperature of T_i=10 keV with stable compression of magnetized plasma by liquid metal liner in repeatable manner. The goal of present study is to use the developed 1D code to search for optimal design point of GF Prototype device by performing scan of plasma and liner parameters.

No-plasma Compression

First, we explore the rotational stabilization of liner compression without plasma. Assumptions:

- incompressible liner is made of liquid lithium with density ρ_0=516 kg/m^3
- part of liner between r_0=1.5 m and r_{rot} is rotating with initial angular velocity Ω_0
- outer surface of liner is pushed by driving gas at constant pressure P_{out}=200 atm
- initial shaft current I_s creates toroidal field, there is no poloidal field in the system
- toroidal flux is conserved during compression, corresponding magnetic pressure is applied to inner surface of liner
- inner surface of liner is stabilized against Rayleigh-Taylor instability if [2] $\ddot r_{in} < \Omega_{in}^2 r_{in}$

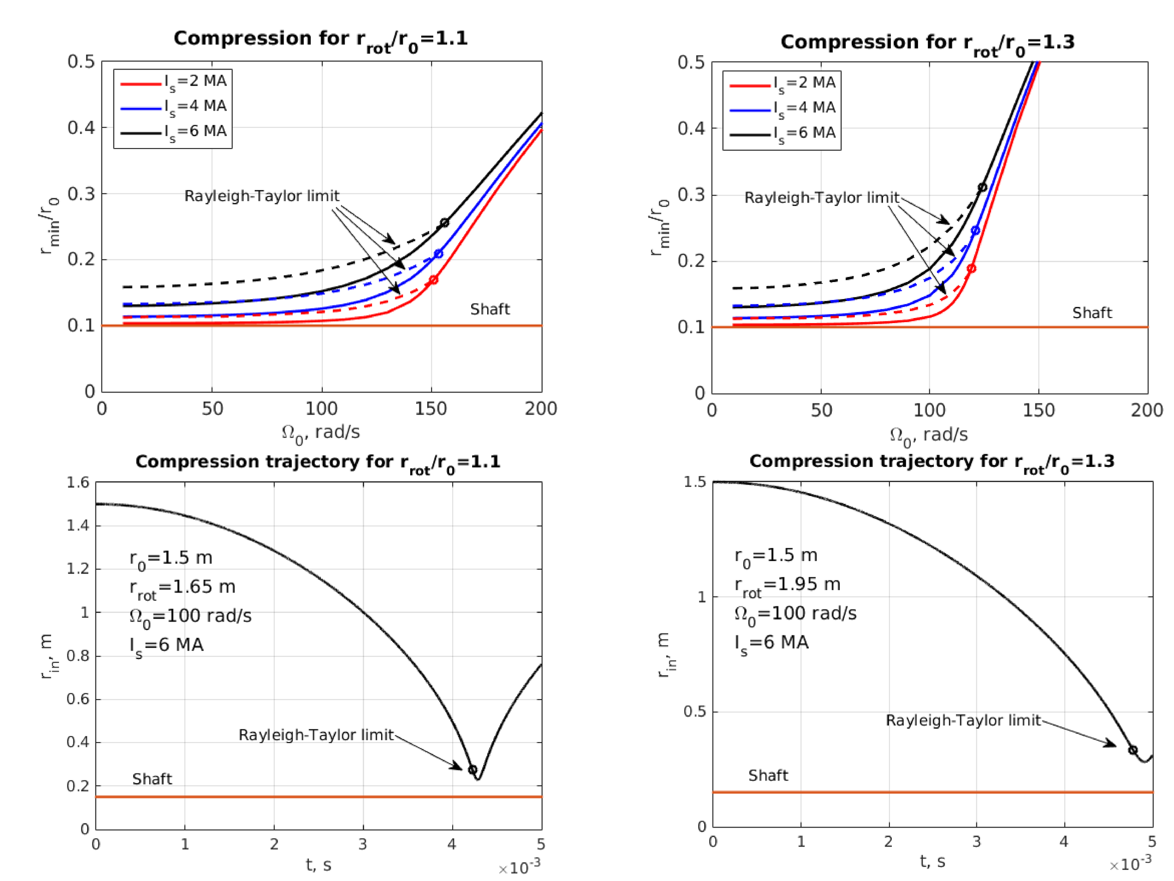

Figure 3: Rayleigh-Taylor stability limits and no-plasma compression trajectories.

Plasma Compression

Scan of multi-parameter Prototype physical space is currently a research in progress. In Fig. 4 we present samples of obtained results.

Assumptions:

- initial plasma Grad-Shafranov equilibrium has shaft current and total poloidal flux that scale with initial cavity radius r_0 as (this makes safety factor *q*>1 everywhere)

$$I_s[\mathrm{MA}] = 1.3 r_0[\mathrm{m}], \qquad \Psi_0[\mathrm{Wb}] = 0.134 r_0^2[\mathrm{m}]$$

- initial peak ion and electron temperatures are $T_{0e}=T_{0i}$=200 eV
- initial density is uniform $n_0=10^{13}$ cm^{-3}
- at start of compression part of liquid lithium liner between r_0 and $r_{rot}=1.1r_0$ is rotating with angular velocity Ω_0=100 rad/s

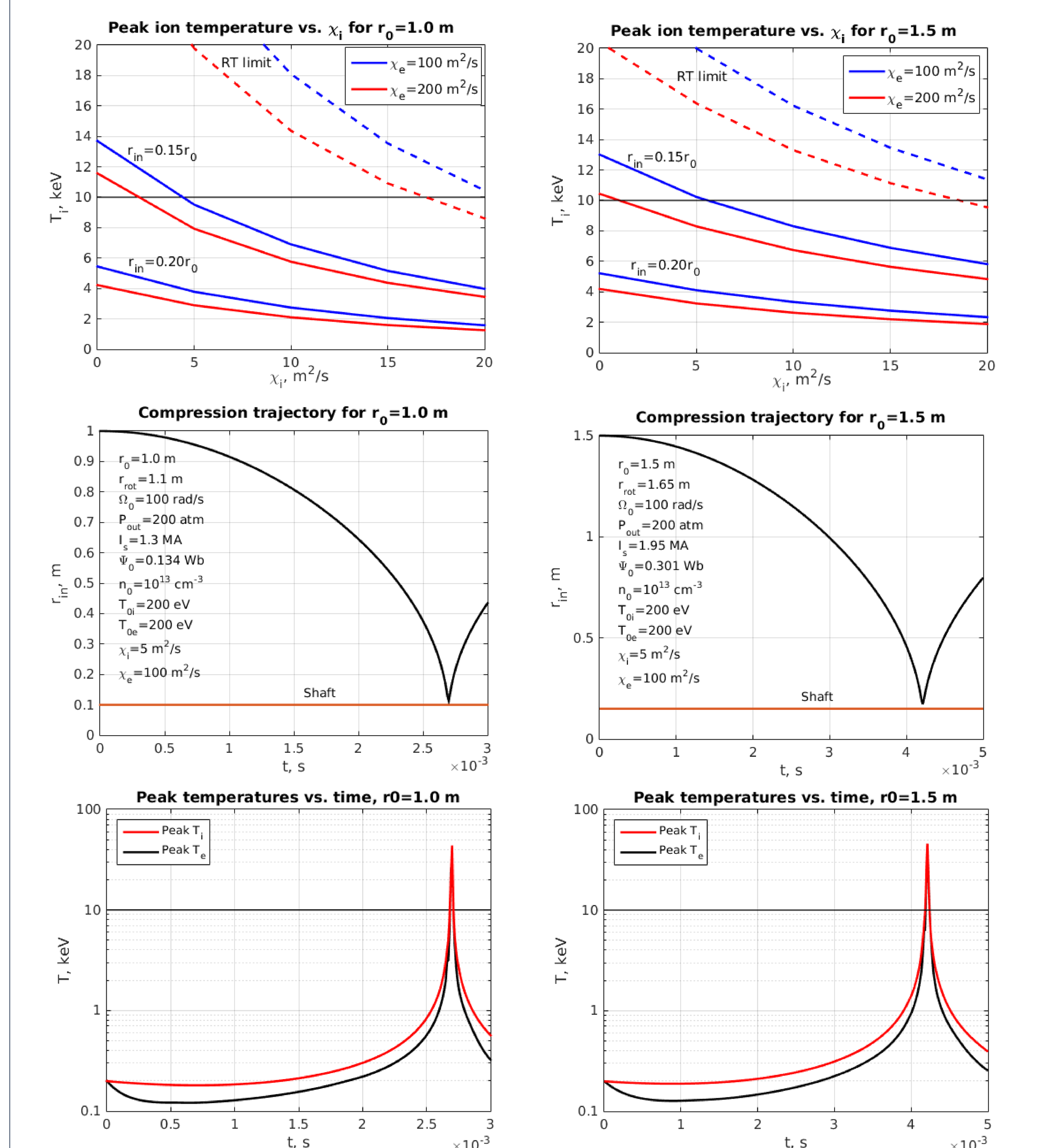

Figure 4: Results of physical parameters scans for Prototype plasma compression.

The parameters scans show strong dependence of achievable ion temperature at compression on thermal diffusivities of both ions and electrons.

For considered range of parameters, our results suggest that peak ion temperature of T_i=10 keV can be obtained by performing ~7x radial compression of plasma with original radius r_0=1.5 m if χ_i<5 m^2/s and χ_e<200 m^2/s.

CONCLUSIONS

- 1D MHD code was developed in General Fusion for simulating coupled dynamics of plasma and liquid metal liner in MTF systems of certain geometry.
- The code was validated against experimental data. In particular, thermal diffusion coefficients and resistivity in physical model were adjusted to match magnetic field and temperature measurements.
- The code was used to scan the space of physical parameters to search for optimal design point of GF Prototype device. This is still work in progress.
- Under reasonable physical assumptions about plasma properties (based on comparison with experiments), our preliminary results suggest that ion temperature of 10 keV can be achieved in 7x radial compression of plasma with initial radius of 1.5 m.
- Because of 1D nature, the code does not capture many important effects present in real plasma-liner system such as deviation of liner from spherical shape, difference of forces in polar and equatorial regions, etc. Eventually GF Prototype project will require the development of (or use of existing, if available) 2D code for the liner-plasma simulations.

REFERENCES

1) Ya. B. Zeldovich, Yu. P. Rayzer "Physics of shock waves and high-temperature hydrodynamic phenomena", Moscow: Nauka, 1966
2) P.J. Turchi "Imploding liner compression of plasma: concepts and issues", IEEE Transactions on Plasma Science, Vol. 36, No. 1, 2008