MHD Simulation Of Plasma Compression Experiments

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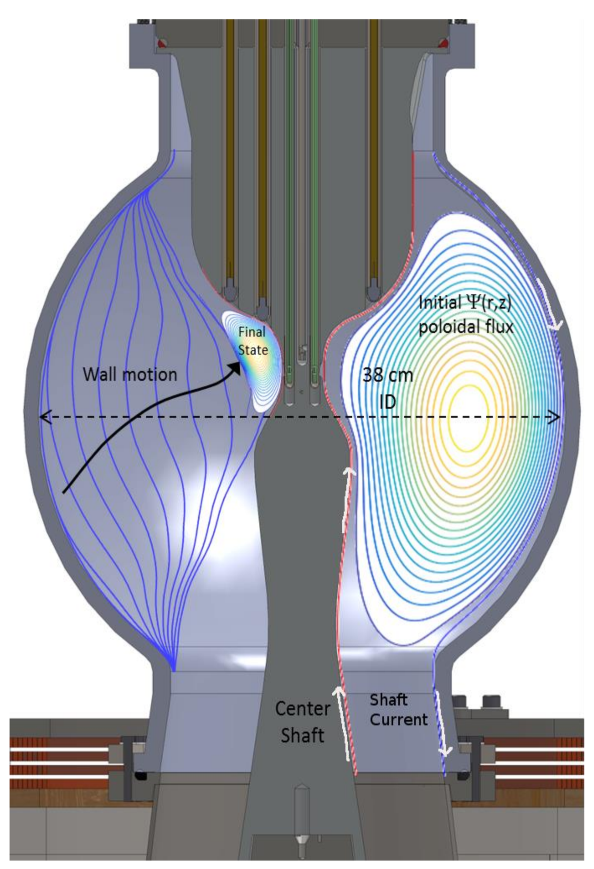
INTRODUCTION

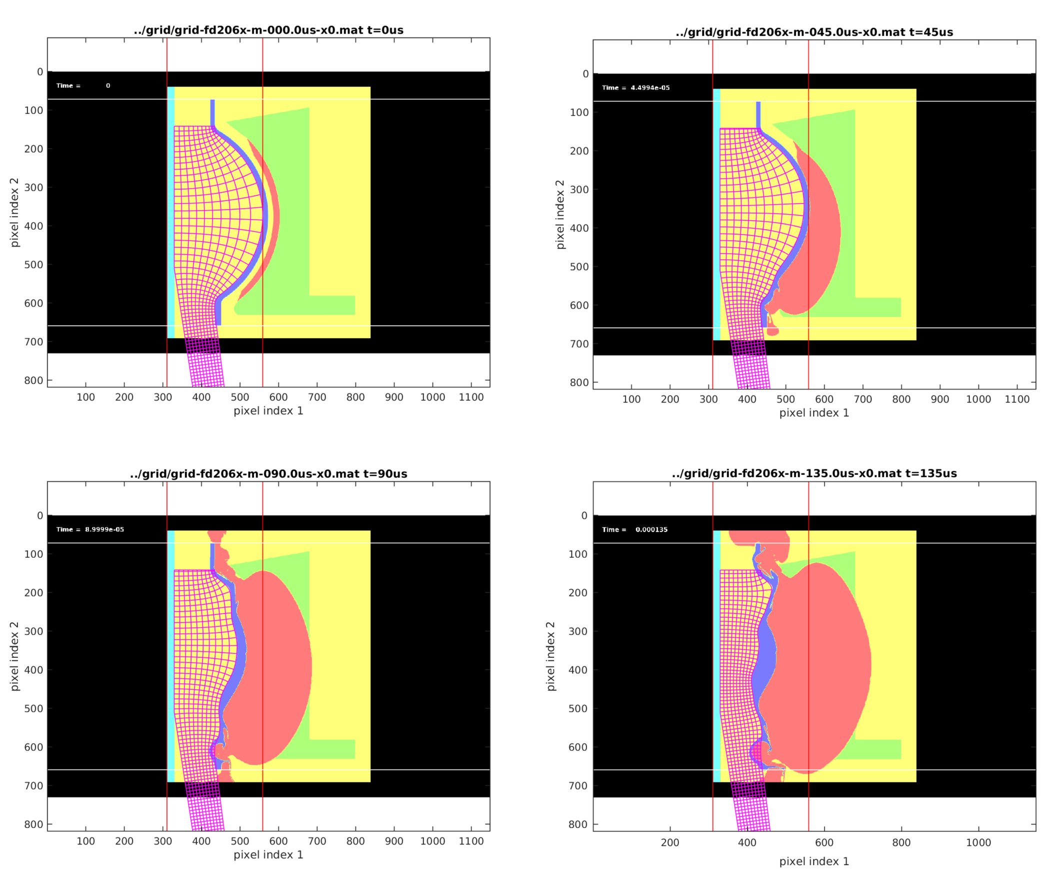
General Fusion (GF) is working to build a magnetized target fusion (MTF) power plant based on compression of magnetically-confined plasma by liquid metal. GF is testing this compression concept by collapsing solid aluminum liners onto plasmas formed by coaxial helicity injection in a series of experiments called PCS (Plasma Compression, Small).

Cross section of SPECTOR PCS experiment



- Shaft current (white arrows) provides toroidal field
- Mirnov probes (colored dots) measure poloidal and toroidal fields
- Inner electrode (Center Shaft) is shaped for 4:1 compression

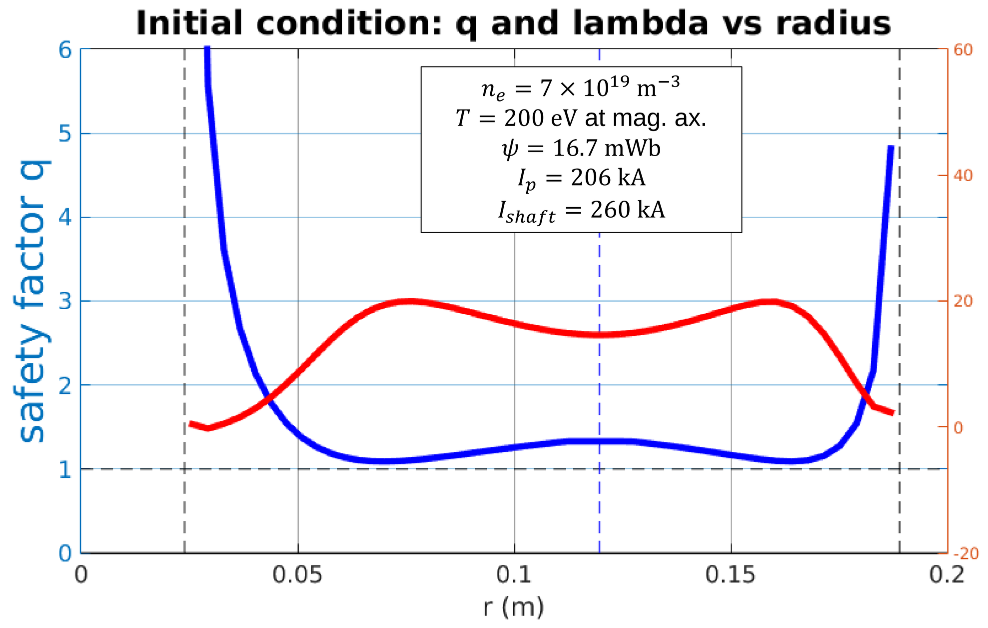
LS-DYNA Liner Trajectory



- Eulerian VOF calculation
- Johnson-Cook model for aluminum 6061 liner (blue)
- Jones-Wilkins-Lee equation of state for chemical driver (red)
- Parameters tuned based on field tests

MHD time-dependent mesh is generated from smoothed LS-DYNA result

Initial condition: q and lambda vs radius



MHD SIMULATION WITH VAC

Shock capturing Eulerian Finite Volume code by Gábor Tóth.

In-house modifications:

- Improvements for strong toroidal fields (e.g., slope-limiting rB_ϕ instead of B_ϕ)
- Coupling MHD to external circuit models
- Independent ion and electron temperatures
- Classical parallel heat transport

Transport:

- Spitzer temperature dependent resistivity
- Various models for radial heat transport, χ
- Constant viscosity for simplicity

Equations of the model

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\mathbf{v}\rho)$$

$$\frac{\partial (\rho\mathbf{v})}{\partial t} = -\nabla\cdot(\mathbf{v}\rho\mathbf{v} - \mu_0^{-1}\mathbf{B}\mathbf{B}) - \nabla p_* + \nabla\cdot\boldsymbol{\Upsilon}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\cdot(\mathbf{v}\mathbf{B} - \mathbf{B}\mathbf{v}) - \nabla\times\mathbf{E}' + \mathbf{e}_\varphi f(r,z) V_{\rm gun}(t)$$

$$\frac{\partial e_{\rm th,e}}{\partial t} = -\nabla\cdot(\mathbf{v}e_{\rm th,e}) - (\gamma-1)e_{\rm th,e}\nabla\cdot\mathbf{v} + G_{\rm ei} + \mathbf{E}'\cdot\mathbf{J} - \nabla\cdot\left(\frac{\mathbf{B}}{|\mathbf{B}|}q_{\|,\rm e} - \kappa_{\perp,\rm e}\nabla(kT_{\rm e})\right)$$

$$\frac{\partial e_{\rm th,i}}{\partial t} = -\nabla\cdot(\mathbf{v}e_{\rm th,i}) - (\gamma-1)e_{\rm th,i}\nabla\cdot\mathbf{v} - G_{\rm ei} + \boldsymbol{\Lambda}:\boldsymbol{\Upsilon} - \nabla\cdot\left(\frac{\mathbf{B}}{|\mathbf{B}|}q_{\|,\rm i} - \kappa_{\perp,\rm i}\nabla(kT_{\rm i})\right)$$

$$\frac{\partial q_{\|,\rm e}}{\partial t} = -\nabla\cdot(\mathbf{v}q_{\|,\rm e}) - \frac{5}{2}n_{\rm e}\frac{kT_{\rm e}}{m_{\rm e}}\frac{\mathbf{B}}{|\mathbf{B}|}\cdot\nabla(kT_{\rm e}) - \frac{q_{\|,\rm e}}{\tau_{q,\rm e}}$$

$$\frac{\partial q_{\|,\rm i}}{\partial t} = -\nabla\cdot(\mathbf{v}q_{\|,\rm i}) - \frac{5}{2}n_{\rm i}\frac{kT_{\rm i}}{m_{\rm i}}\frac{\mathbf{B}}{|\mathbf{B}|}\cdot\nabla(kT_{\rm i}) - \frac{q_{\|,\rm i}}{\tau_{q,\rm i}}$$

$$p_* = p + \frac{B^2}{2\mu_0} \quad ; \quad p = (\gamma-1)(e_{\rm th,e} + e_{\rm th,i}) \quad ; \quad \gamma = 5/3$$

$$\mathbf{J} = \mu_0^{-1}\nabla\times\mathbf{B} \quad ; \quad \mathbf{E}' = \eta\mathbf{J}$$

$$\boldsymbol{\Upsilon} = 2\mu\boldsymbol{\Lambda} \quad ; \quad 2\boldsymbol{\Lambda} = (\nabla\mathbf{v}) + (\nabla\mathbf{v})^\top - \frac{1}{3}{\rm Tr}\left[(\nabla\mathbf{v}) + (\nabla\mathbf{v})^\top\right]$$

Compression is much slower than plasma dynamics

Physics	Time scale	Velocity
Compression	$\tau_{\rm compr} \simeq 130\,\mu s$	$v_{\rm compr} \simeq 1.5\times10^3\,{\rm m/s}$
Plasma sound	$\tau_{\rm s} \simeq 1.2\,\mu s$	$c_s \simeq 10^5\,{\rm m/s}$
Alfvén wave	$\tau_{\rm A} \simeq 0.1\,\mu s$	$v_{\rm A} \simeq 10^6\,{\rm m/s}$

Time scale ordering:

$$\tau_{\rm compr} \gg \tau_{\rm s} > \tau_{\rm A}$$

∴ Use quasi-static approximation to implement compression.

Every 100 time steps (1-10 ns) do the following:

First, transform physical quantities to compression invariants:

Invariant	Conserved quantity
$\sqrt{g}\rho$	mass
$\sqrt{g}\rho v_i$	angular momentum
$\sqrt{g}B^i$	magnetic flux
$p/\rho^{5/3}$	entropy

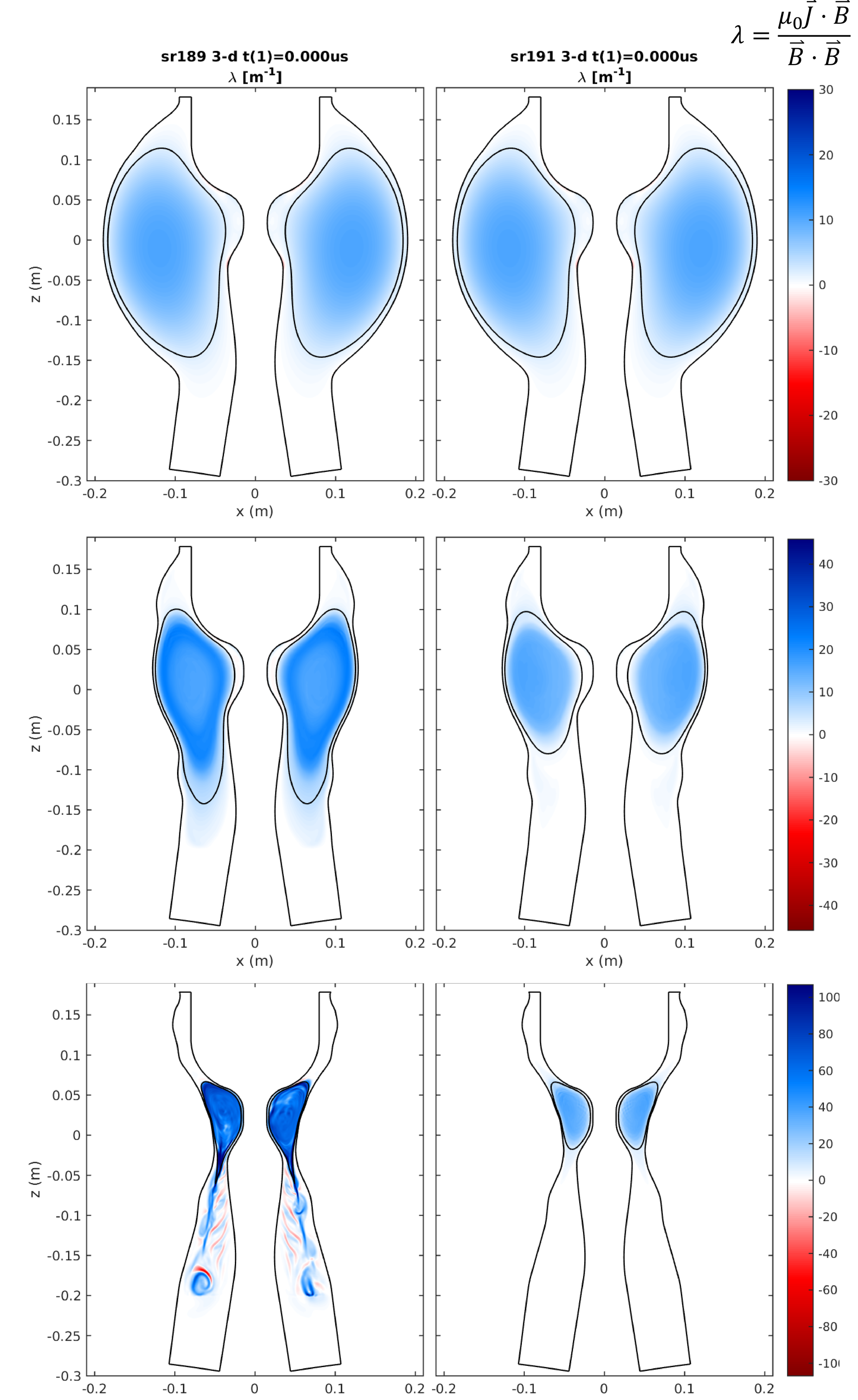
(tensors with respect to logical coordinates, $\sqrt{g}$ is cell volume).

Next, update mesh geometry, replacing physical coordinates.

Last, transform back to physical quantities using new relationship between physical and logical coordinates.

STABILIZATION BY RAMPING SHAFT CURRENT

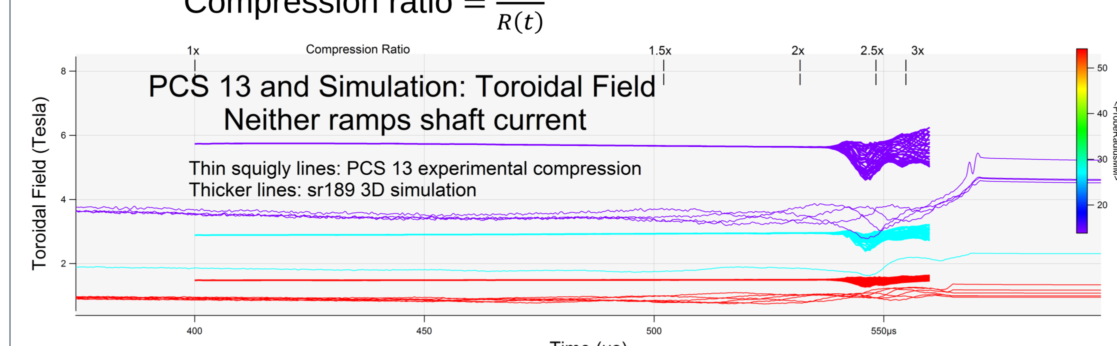
$$\lambda = \frac{\mu_0 \vec{J}\cdot\vec{B}}{\vec{B}\cdot\vec{B}}$$



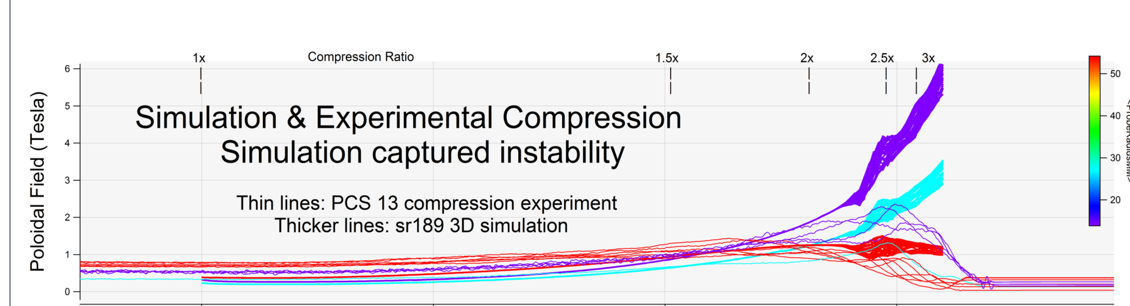
Constant shaft current: $I_{shaft}(t) = I_0$

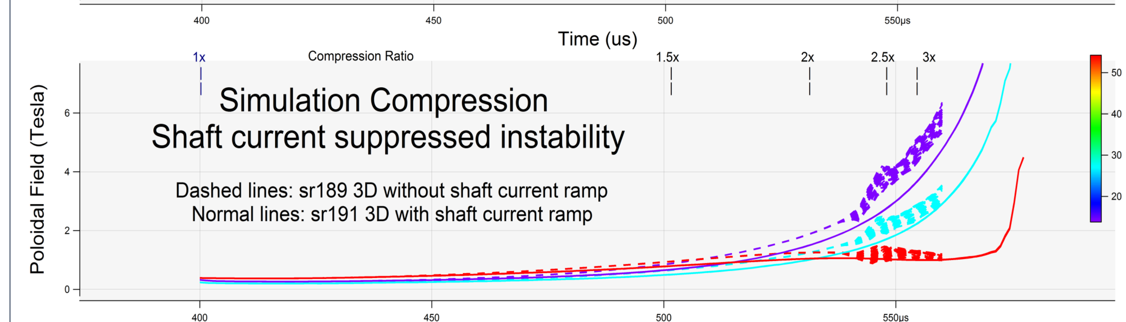
Ramped shaft current: $I_{shaft}(t) \propto \frac{1}{r(t)}$

Compression ratio $= \frac{R_0}{R(t)}$

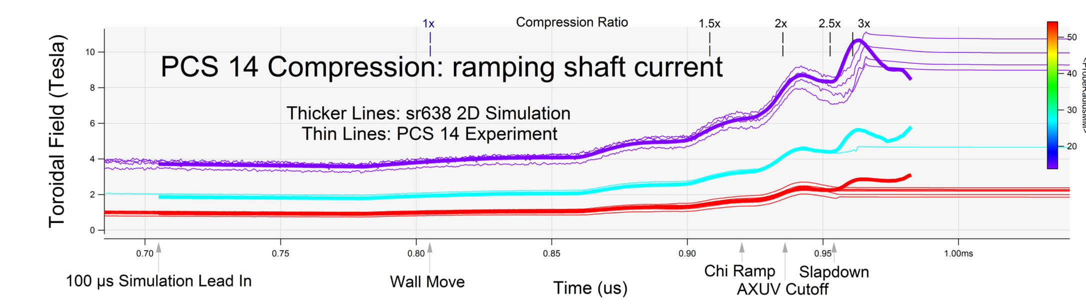


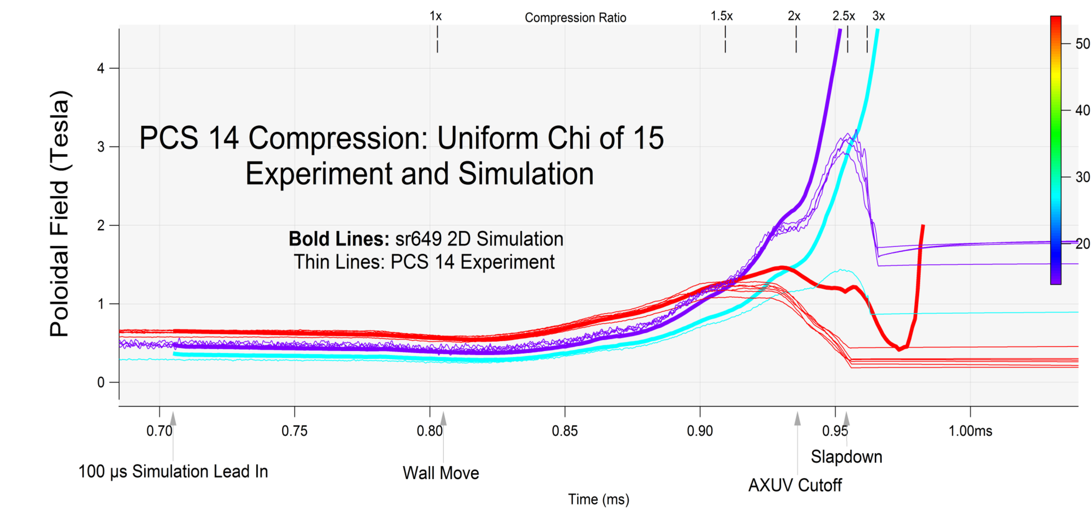
$$\mu_0 I = 2\pi r B_\phi$$



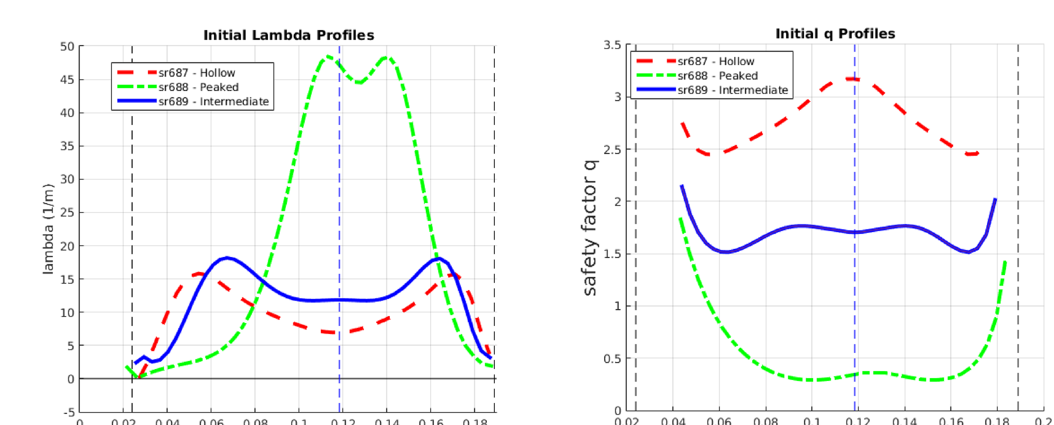


SIMULATION OF PCS14

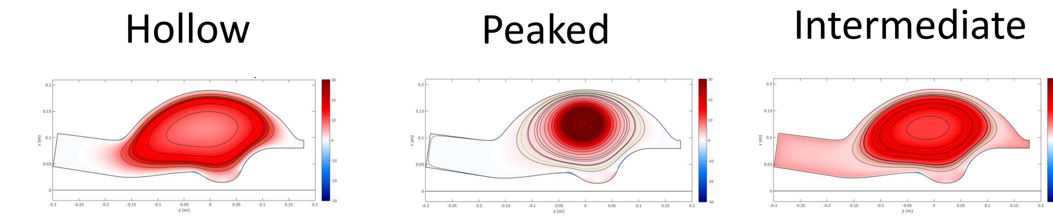


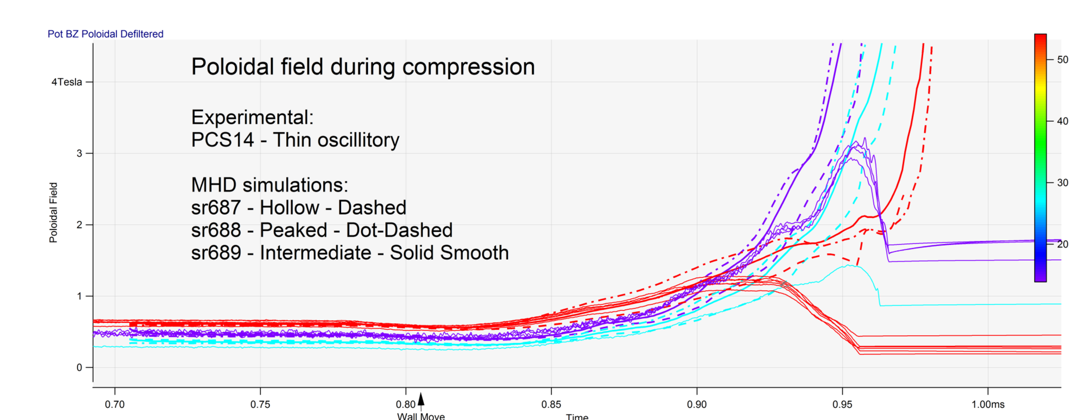


SENSITIVITY TO CURRENT PROFILE



Hollow | Peaked | Intermediate





SUMMARY

Shaft current ramp:

- MHD simulations showed stabilizing effect
- motivated inclusion in PCS14 experiment
- compression was stable at least to R0/R = 2.5x

Modeling PCS14:

- MHD simulation initialized to conditions of PCS14
- Matches decay of plasma current prior to compression
- Matches compression increase of plasma current until a compression ratio of about 1.7x, then experiment falls below simulation.

Comparing current profiles for compression:

- Very different current profiles yield qualitatively similar results
- Matching experiment will require additional phenomena

ACKNOWLEDGEMENTS

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