Modeling Highly Unsteady Current-driven Liquid Metal Free-Surface MHD Flows

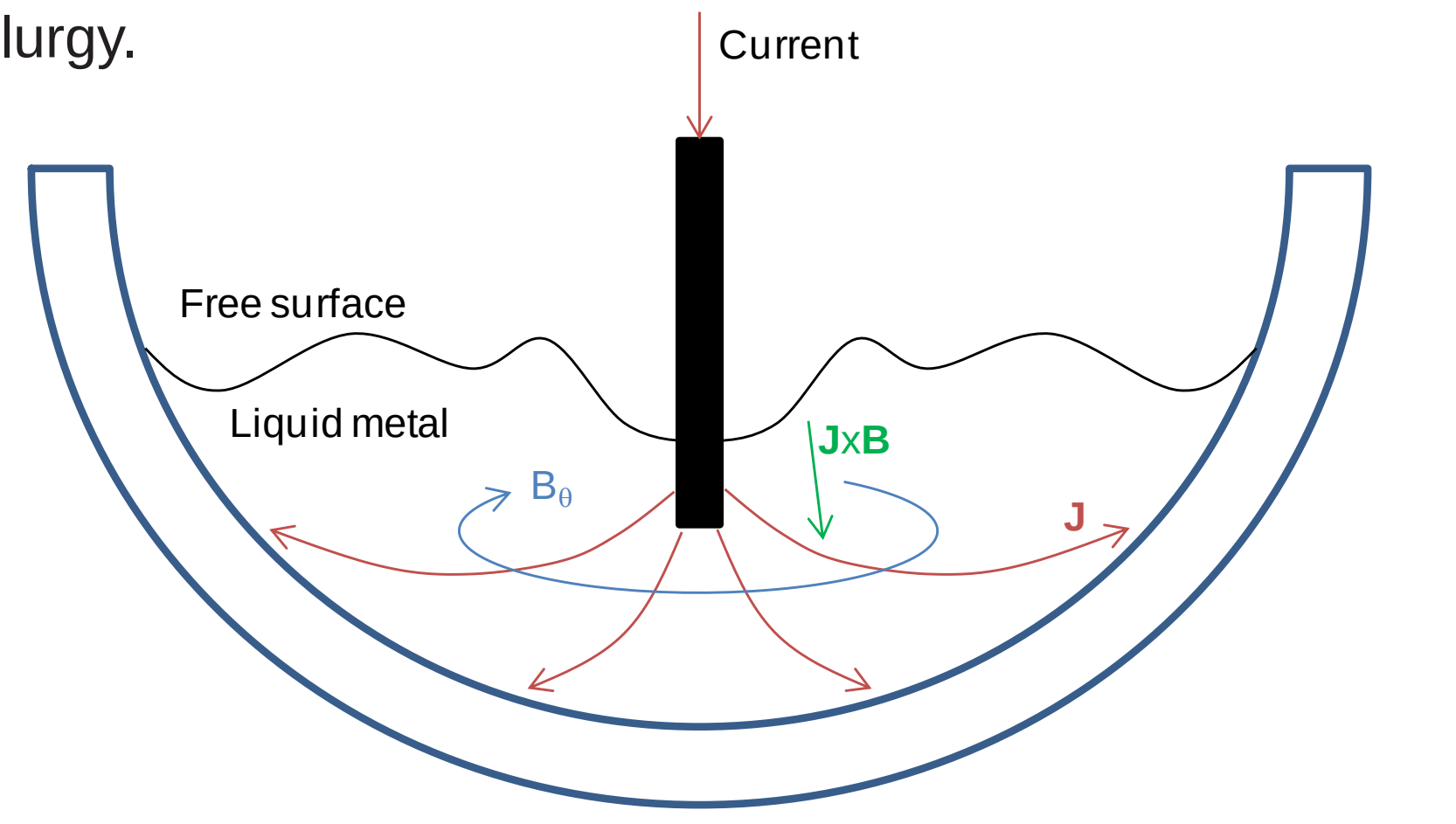
Ramakanth Munipalli[1], Peter Huang[1], Rupanshi Chhabra[1], Alex Mossman[2],
Stephen Howard[2], Wade Zawalski[2], Meritt Reynolds[2], David Plant[2]

[1]HyPerComp Inc., Westlake Village, California, USA [2]General Fusion Inc., Burnaby, British Columbia, Canada

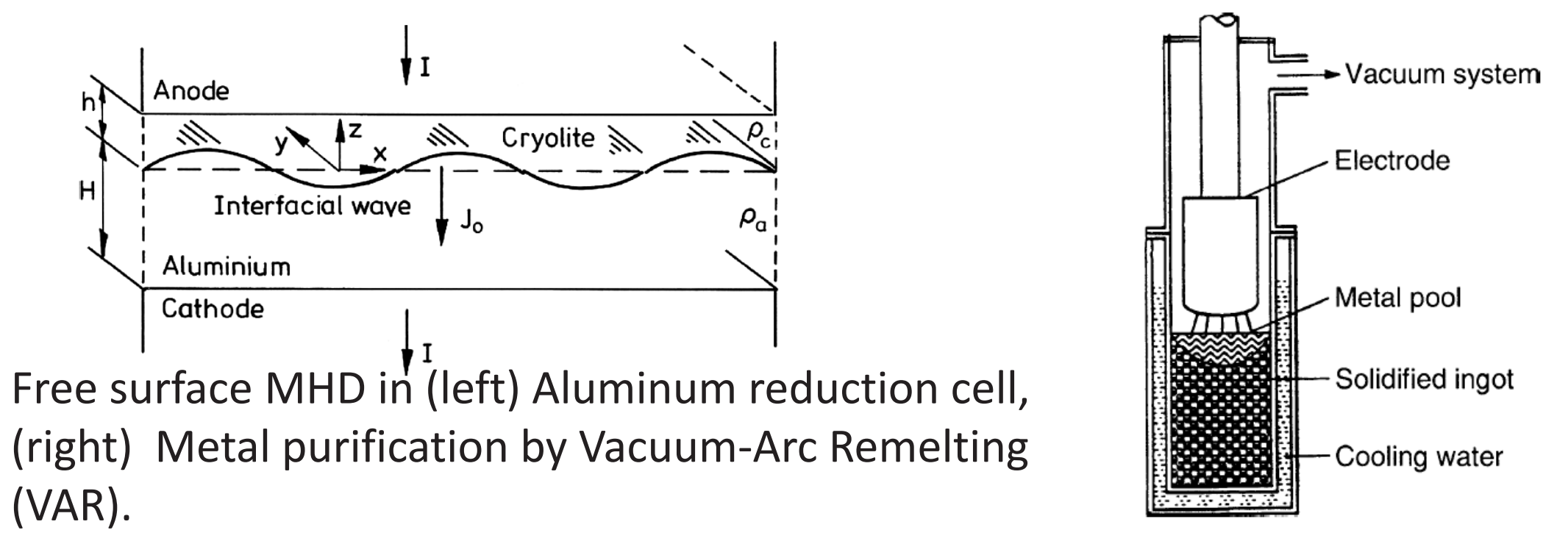
60th Annual Meeting of the APS Division of Plasma Physics, Portland, Oregon, November 5-9, 2018 **DPP18-001282**

MHD Flows Driven by Unsteady Current

The flow of electrically conducting liquids in response to a highly unsteady applied current (or magnetic field) is of practical interest in nuclear fusion, and metallurgy.



Current enters a pool of liquid metal from an electrode, induces a toroidal magnetic field (in the above case), resulting in a Lorentz JxB force which drives the liquid metal pool.



Free surface MHD in (left) Aluminum reduction cell, (right) Metal purification by Vacuum-Arc Remelting (VAR).

Images from Davidson, P.A., "Intro to MHD" 2nd ed., 2017

There are very limited demonstrations of comprehensive (self-consistent) modeling for such flows in published literature at present. We developed a parallel, unstructured mesh solver to extend methods commonly used in eddy current analysis to current (or voltage) driven MHD simulations.

This effort is motivated by the General Fusion SLiC experiment. SLiC will facilitate testing of liquid lithium compatible plasma diagnostics and further our understanding of plasma dynamics due to interactions with liquid lithium. (more details in the "SLiC Application" section)

HyPerComp Inc. - The HIMAG Code

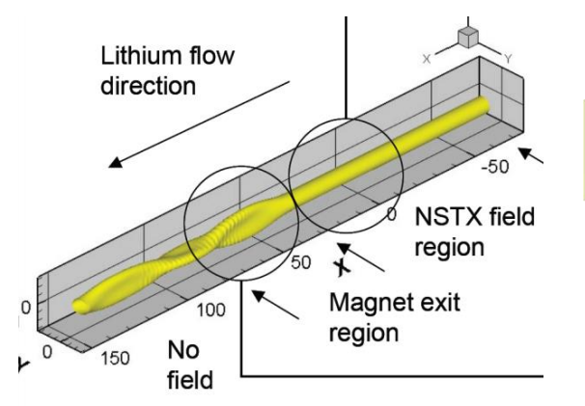
HyPerComp Incompressible MHD solver for Arbitrary Geometry

HIMAG is a pioneering complex geometry modeling software for incompressible two-phase magnetohydrodynamics (MHD) flows at fusion-relevant conditions.

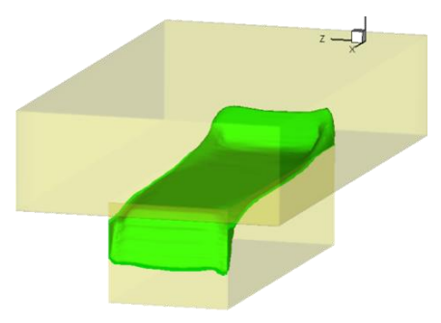
HIMAG is used to model the flow of liquid metals and other flows occurring in fusion reactors where strong interactions with the magnetic field produce strongly coupled effects that are unique to MHD.

Summary of HIMAG capabilities

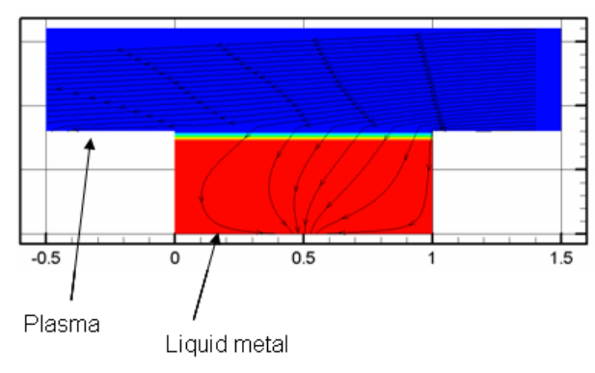
- Three dimensional incompressible flow solver (second order accurate in space and time)
- Free surface capture using the level set method
- Arbitrary mesh structure (hexahedral / tetrahedral / prismatic cells)
- Well tested highly parallel code environment
- Electric potential as well as induced magnetic field formulations for MHD
- Point implicit scheme, solved in an iterative manner
- Multiple strategies to account for mesh skewness (non-orthogonality)
- Multiple solid walls of different conductivity and contact resistance
- Heat transfer and natural convection, temperature dependent properties

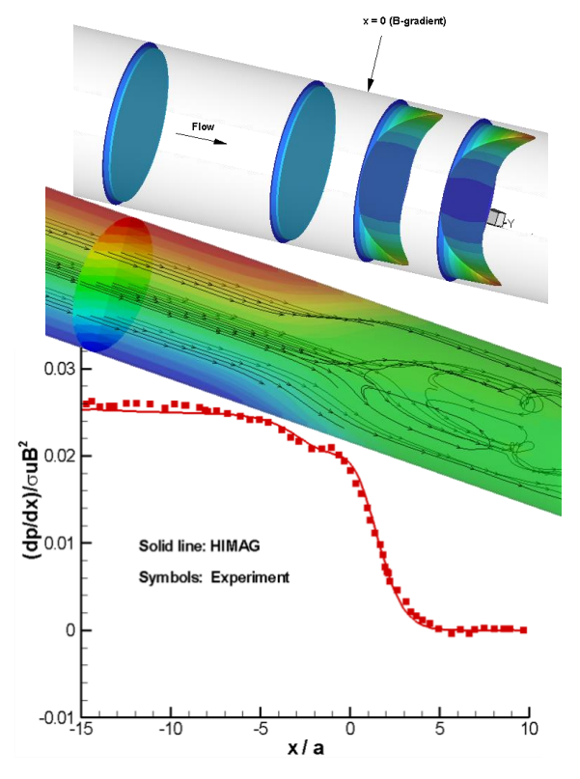


Liquid metal jet in a magnetic field

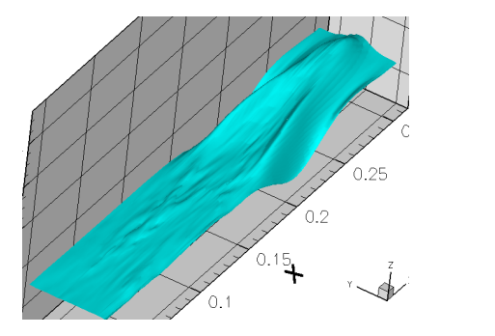


Liquid metal in a magnetic field, sloshing (above) in response to electric current entering from an adjoining plasma (below)





Flow in a circular duct at a Hartmann number of 6640, emerging from a region of strong magnetic field. First of a kind demonstration of such validation in a full 3D solver at high Hartmann numbers

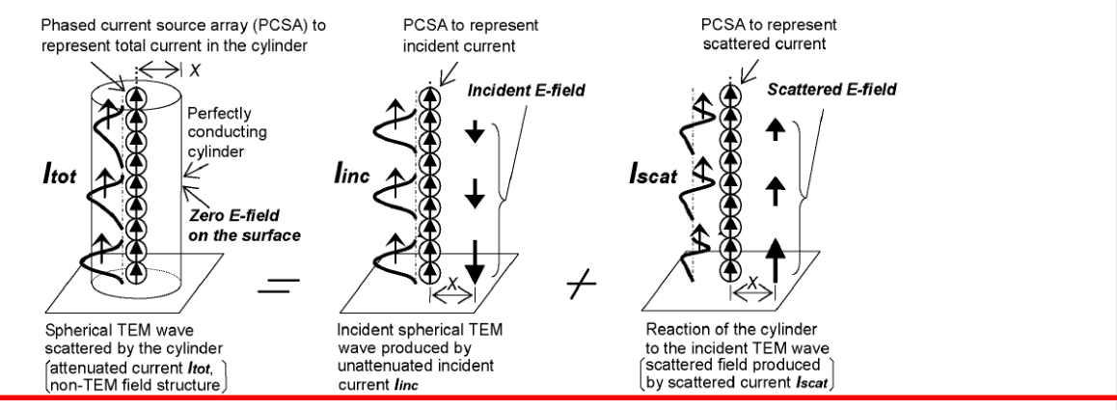


Free surface thickening due to MHD effects in an open channel flow.

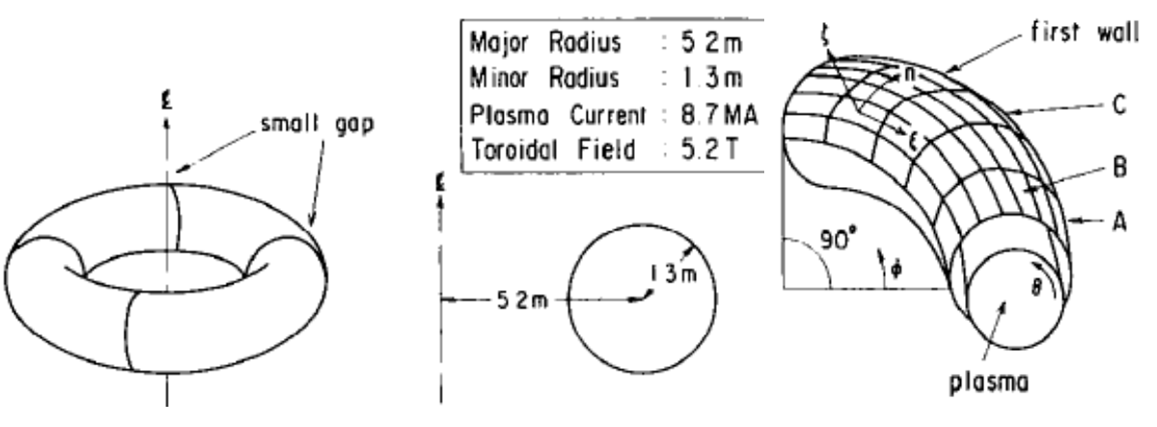
HIMAG was developed under funding from the U.S. Department of Energy. For availability and enquiries concerning MHD and free surface modeling research at HyPerComp, please email: contact@hypercomp.net

Comparable Models

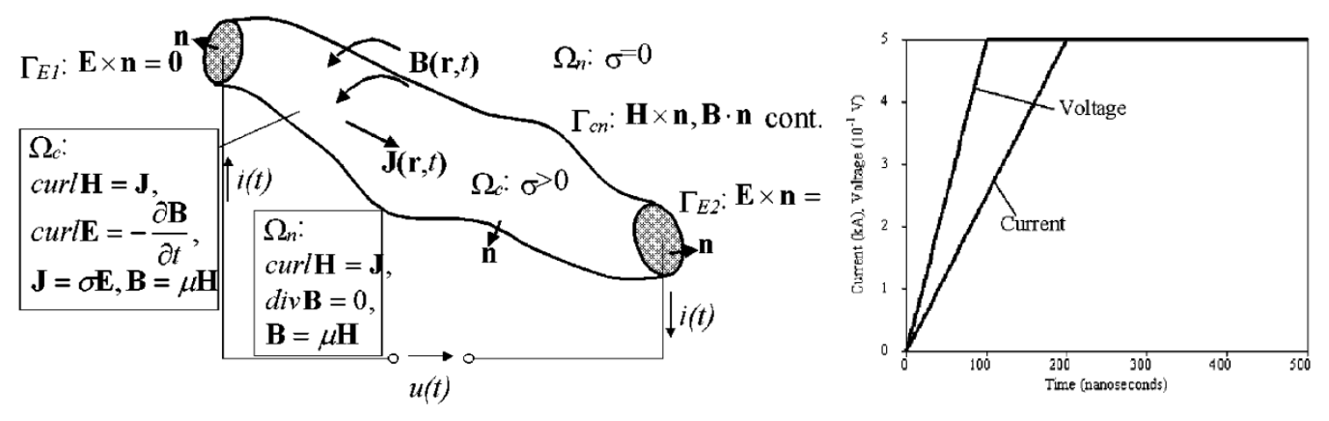
Full wave EM, and eddy current models have been used in related studies



Lightning:
Baba, Y., Rakov, V.A., "Electromagnetic models of the lightning return stroke," J. Geophys. Res., V. 112, D04102, 2007



Plasma Disruption:
Hashizume, H., Yoshida, Y., Miya, K., Ioki, K., "Magnetothermohydraulic behavior of the molten layer of first wall due to plasma disruption," Fus. Eng. Des., V. 9 pp. 219-224, 1989
Kameari, A., "Transient eddy current analysis on thin conductors with arbitrary connections and shapes," J. Comp. Phys., V. 42, pp. 124-140, 1981



Skin Effects in Power Systems:
Biro, O., Preis, K., Wachutka, G., "Edge Finite Element Analysis o Effect Problems," IEEE Trans. Magnetics, V. 36, No. 4, pp. 835-83
Bohm, P., Wachutka, G., "Numerical Analysis Tool for Transient S Problems," 35th IEEE Power Electronics Specialists Conference, 2

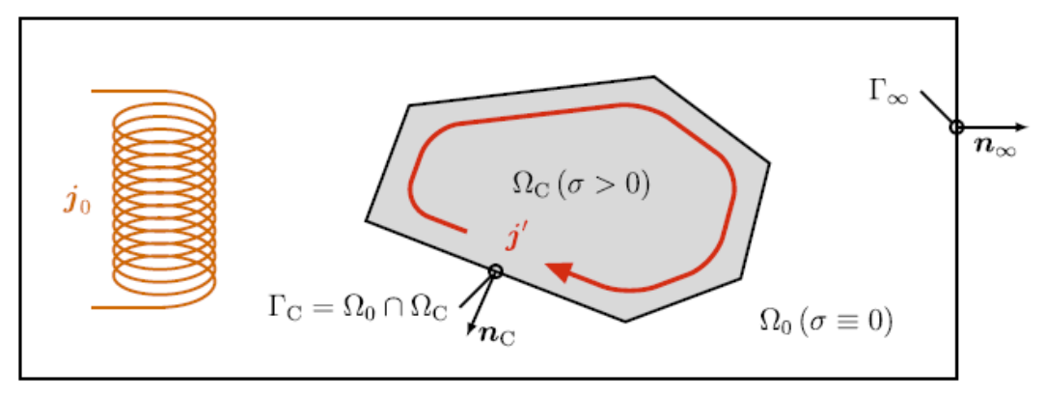
Different models used in eddy current literature

Formulation	variable location	in the current carrying region	current free region
★ A - ϕ	A ϕ A	curl (ν curl (A)) = -σ (∂A/ ∂ t + grad ϕ)	curl (ν curl (A)) = 0
A - ϕ - Ω	A ϕ Ω	div (-σ (∂A/ ∂ t + grad ϕ)) = 0	div(- μ grad Ω) = 0
A* - Ω	A* Ω	curl (ν curl (A*)) = -σ (∂A/ ∂ t)	
★ T - Ω	T Ω Ω	curl(curl (T)/ σ) = - ∂/∂ t (μ(T - grad Ω)) div (μ(T - grad Ω)) = 0	
E - Ω	E Ω	curl (ν curl (E)) = -σ (∂ E / ∂ t)	

Legend:

A : Magnetic vector potential B = curl (A)
ϕ : Electric scalar potential E = - (∂A/ ∂ t + grad ϕ)
Ω : Magnetic scalar potential H = - grad Ω
A* : Modified A : A* = A + ∫ grad ϕ dt
T : Current vector potential J = curl (T)
μ : mag. perm., ν = 1/ μ, σ = elec. cond.

Separate regions for storing variables : J ≠ 0, J = 0



In eddy current applications, current crossing a material boundary is typically not modeled.

Unique aspects of this work: (**a**) Current and/or voltage driven flows, (**b**) Unsteady formulation – accounts for skin effect, (**c**) Implicit free surface capture allows for large boundary deformation of a conducting liquid (**d**) Formal verification & validation process

Mathematical Formulation

Fluid flow: Incompressible Navier-Stokes equations using the level set method for interface capture, solved using the Crank-Nicholson scheme

$$\frac{\partial \phi}{\partial t} + \nabla \cdot \left(\vec{V} \phi \right) = 0 \quad \text{Level set advection}$$

$$\frac{\partial \varphi}{\partial \tau} = sign(\varphi_0)\left(1 - |\nabla \varphi|\right) \quad \text{Re-initialization}$$

$$\nabla \cdot \vec{V} = 0$$

$$\rho = \rho_2 + (\rho_1 - \rho_2) H_\varepsilon(\phi)$$
$$\mu = \mu_2 + (\mu_1 - \mu_2) H_\varepsilon(\phi)$$
$$\sigma = \sigma_2 + (\sigma_1 - \sigma_2) H_\varepsilon(\phi)$$

Gravity / Buoyancy, Surface tension

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho \left(\vec{V} \cdot \vec{\nabla} \right) \vec{V} = -\nabla p + \mu \nabla^2 \vec{V} + \vec{J} \times \vec{B} + \vec{F}_b + \vec{F}_{ST}$$

Electromagnetics

1. Biot-Savart Law
(easy, but expensive and limited utility)

$$\vec{B} = \frac{\mu_0}{4\pi} \int_V \frac{\left(\vec{J}\, dV \right) \times \vec{r}}{r^3}$$

$$\left(\nabla \cdot \vec{J} = 0 \right)$$

$$\vec{\nabla} \cdot \sigma \left(\vec{\nabla} \varphi \right) = \vec{\nabla} \cdot \sigma \left(\vec{V} \times \vec{B} \right) \longrightarrow \vec{J} = \sigma \left(-\vec{\nabla} \varphi + \vec{V} \times \vec{B} \right) \quad \text{Ohm's law}$$

Electromagnetics

2. Magnetic Vector Potential(more complex, but more general, and numerically efficient)

$$\boxed{\begin{aligned} &\frac{\partial \vec{A}}{\partial t} = -\nabla \varphi + \frac{1}{\mu_0 \sigma} \nabla^2 \vec{A} + \vec{V} \times \vec{B} \\ &\nabla \cdot \vec{A} = 0 \end{aligned}}$$

$$\text{Ohm's law} \quad \vec{J} = \sigma \left(-\nabla \varphi - \frac{\partial \vec{A}}{\partial t} + \vec{V} \times \vec{B} \right)$$

$$= \nabla \times \frac{1}{\mu_0} \nabla \times \vec{A}$$

$$\text{Charge conservation} \quad \nabla \cdot \vec{J} = 0$$

$$\Rightarrow \quad \nabla \cdot \left(\sigma \left(-\nabla \varphi - \frac{\partial \vec{A}}{\partial t} + \vec{V} \times \vec{B} \right) \right) = 0$$

Fractional step scheme
(using Crank-Nicholson discretization)

$$\frac{\hat{A} - \vec{A}^n}{\Delta t} = -\nabla \varphi^n + \frac{1}{\mu_0 \sigma} \nabla^2 \vec{A} + \vec{V} \times \vec{B}$$

$$\frac{\vec{A}^* - \hat{A}}{\Delta t} = \nabla \varphi^n$$

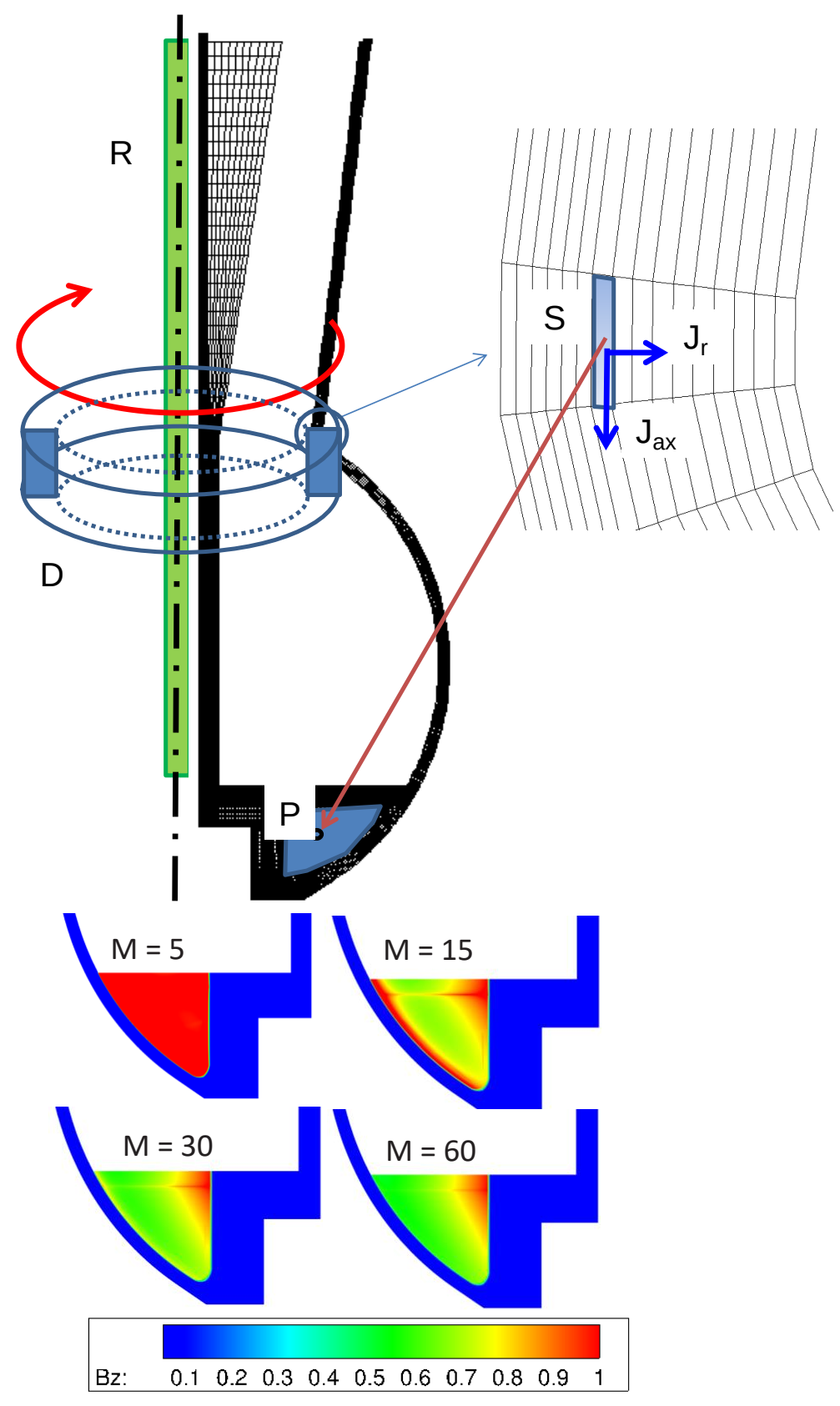
$$\frac{\vec{A}^{n+1} - \vec{A}^*}{\Delta t} = -\nabla \varphi^{n+1}$$

$$\nabla^2 \varphi^{n+1} = \frac{1}{\Delta t} \nabla \cdot \vec{A}^*$$

Boundary conditions: *A* x *n* = 0 on surfaces where current may flow across, specifying *A* or φ as function of time. *J* . *n* = 0 at insulating boundaries. Implicit capture of internal interfaces.

Numerical Method Development

We began with an EM model based on the Biot-Savart law

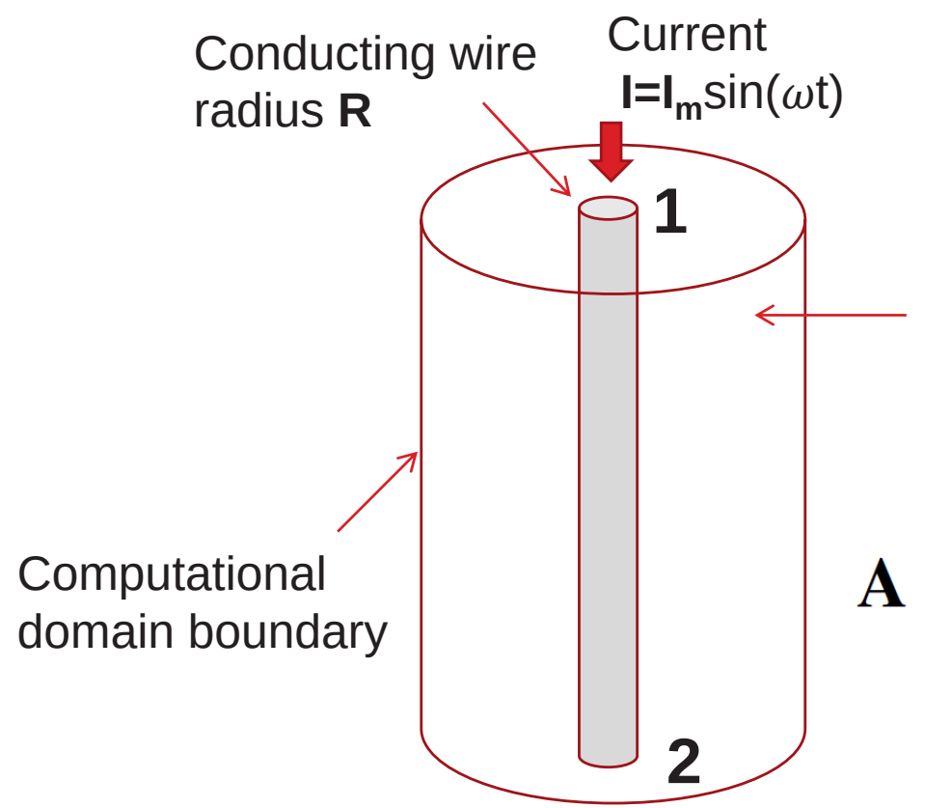


3 Methods to compute induced B using Biot-Savart law, given input current:

1. B induced at all points P in fluid region shown solely due to the green conducting rod 'R' which carries an axial current
2. B is induced at P by 3D volume element sources S each corresponding to a computational cell, bearing radial and axial current densities shown
3. B induced at P due to all of these cells S, but reduce numerical burden by placing P in an a 2D plane that is axially symmetric, and integrating the sources S about the disc region D analytically/numerically (M slices)

Drawbacks: Computationally expensive, cannot model transients, skin effect
This led to the vector potential based model in HIMAG

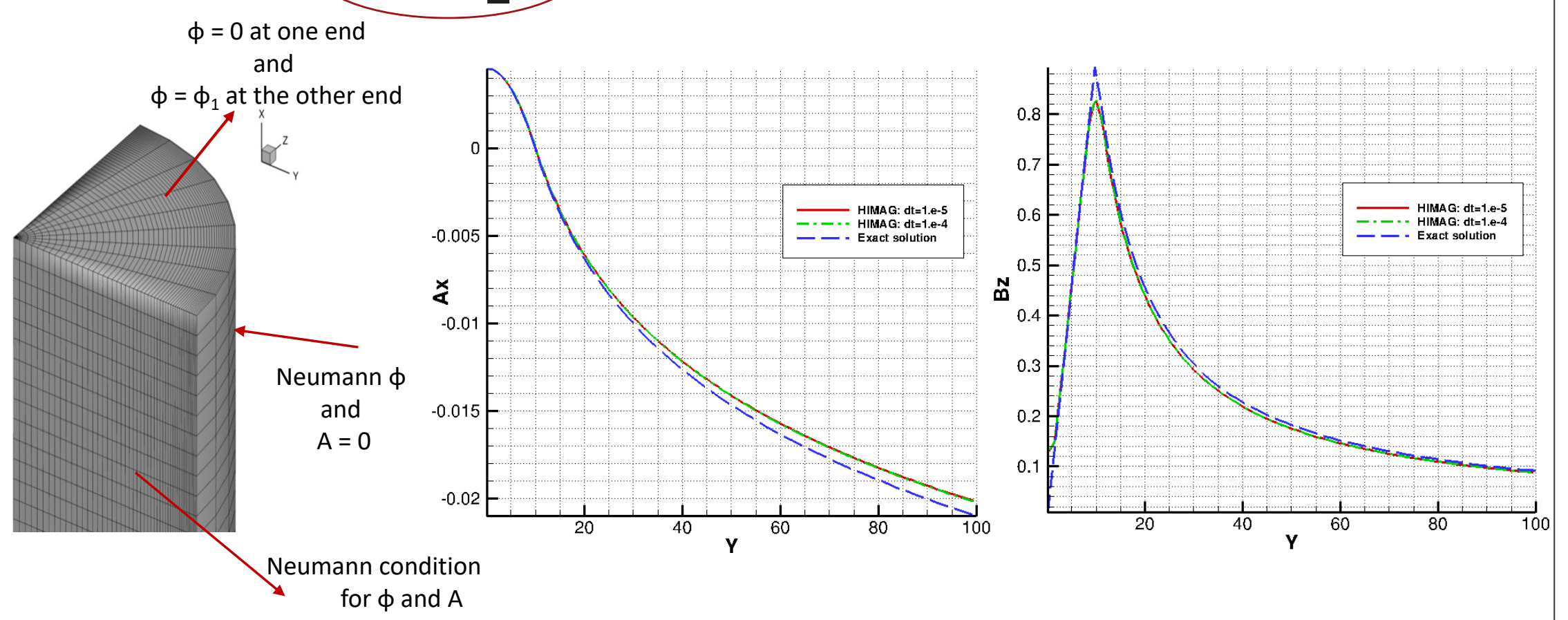
Vector potential formulation: Code Verification



DC Current flow in a cylindrical wire

Analytic solution for an infinite wire with vacuum outside:

$$\mathbf{A} = \begin{cases} -\frac{I\mu_0}{2\pi} \ln \frac{r}{R} & r \geq R \quad \mathbf{B} = \frac{I\mu_0}{2\pi r}\hat{\phi} \\ -\frac{\mu_0 I}{4\pi R^2}\left(r^2 - R^2\right) & r \leq R \quad \mathbf{B} = \frac{\mu_0 r I}{2\pi R^2}\hat{\phi} \end{cases}$$



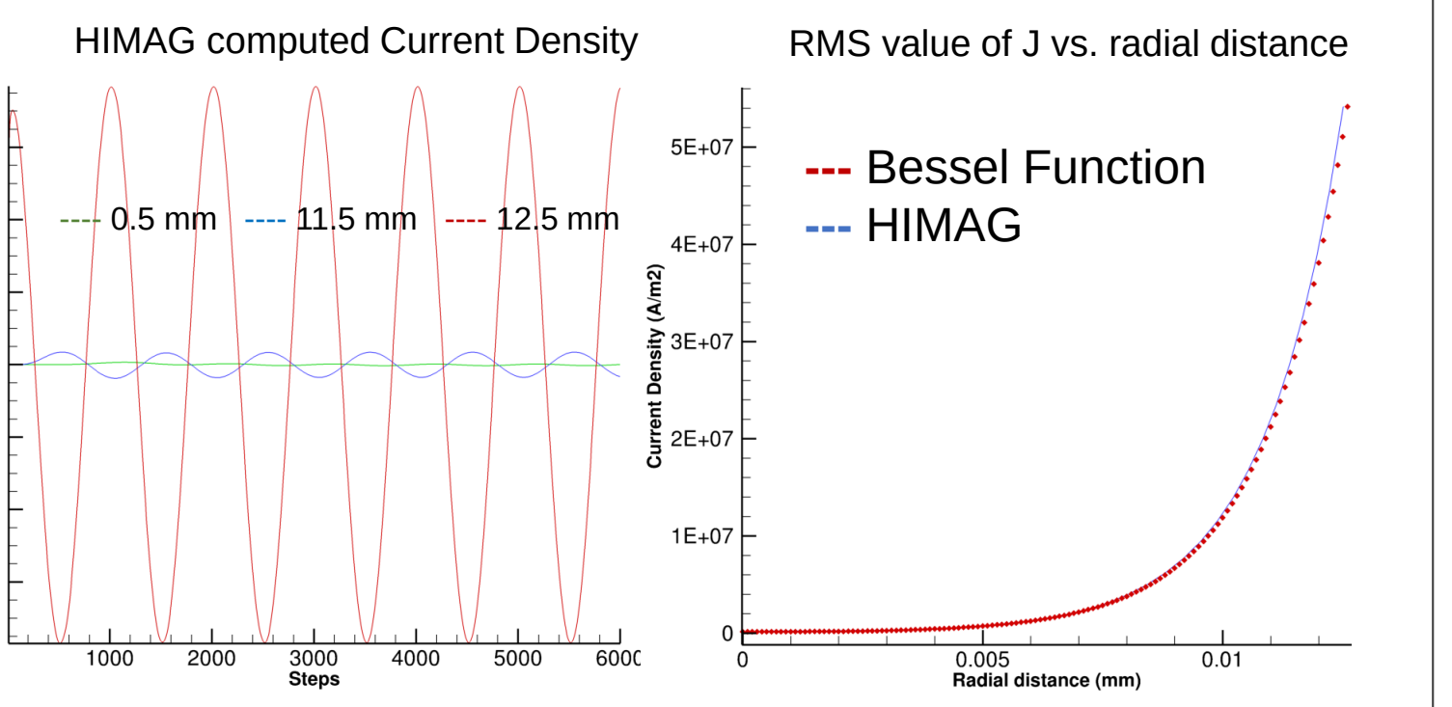
AC Current flow in a cylindrical wire – skin effect

Analytical expression for current

$$\mathbf{J}_r = \frac{k\mathbf{I}}{2\pi R} \frac{J_0(kr)}{J_1(kR)} = \mathbf{J}_R \frac{J_0(kr)}{J_0(kR)}$$

$$k = \sqrt{\frac{-j\omega\mu}{\rho}} = \frac{1-j}{\delta}$$

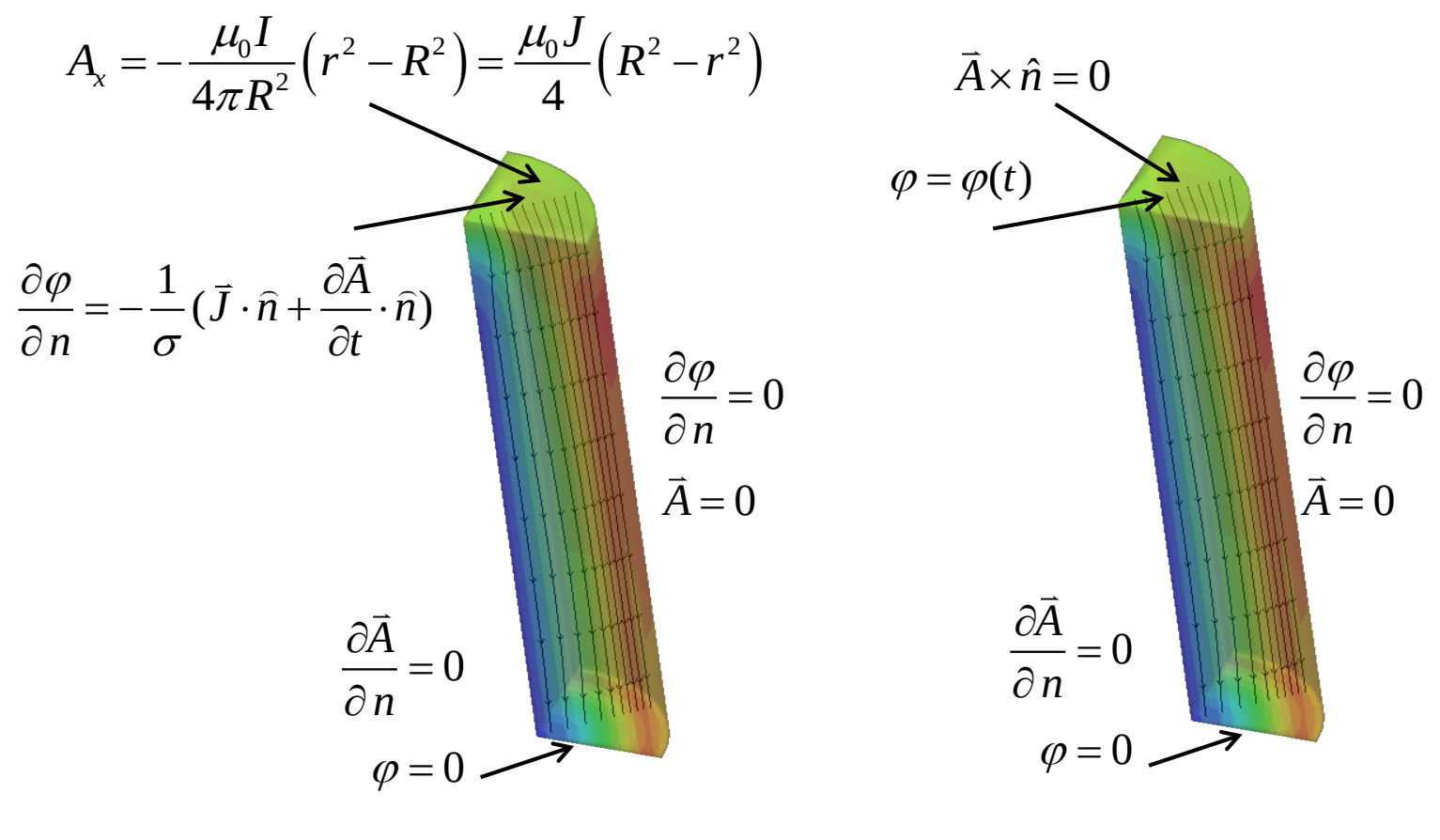
$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$


HIMAG computed Current Density

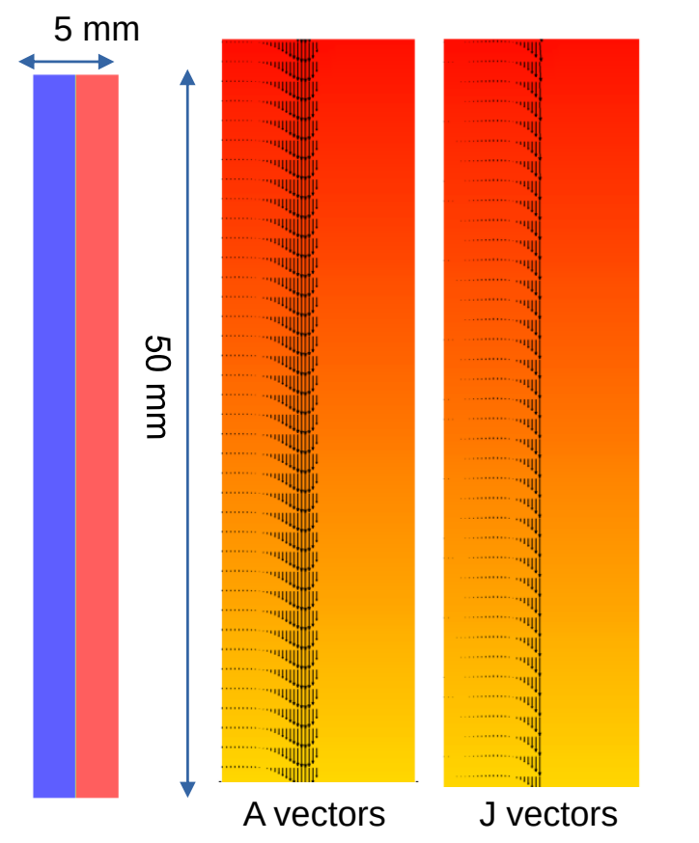
RMS value of J vs. radial distance

Current Feed

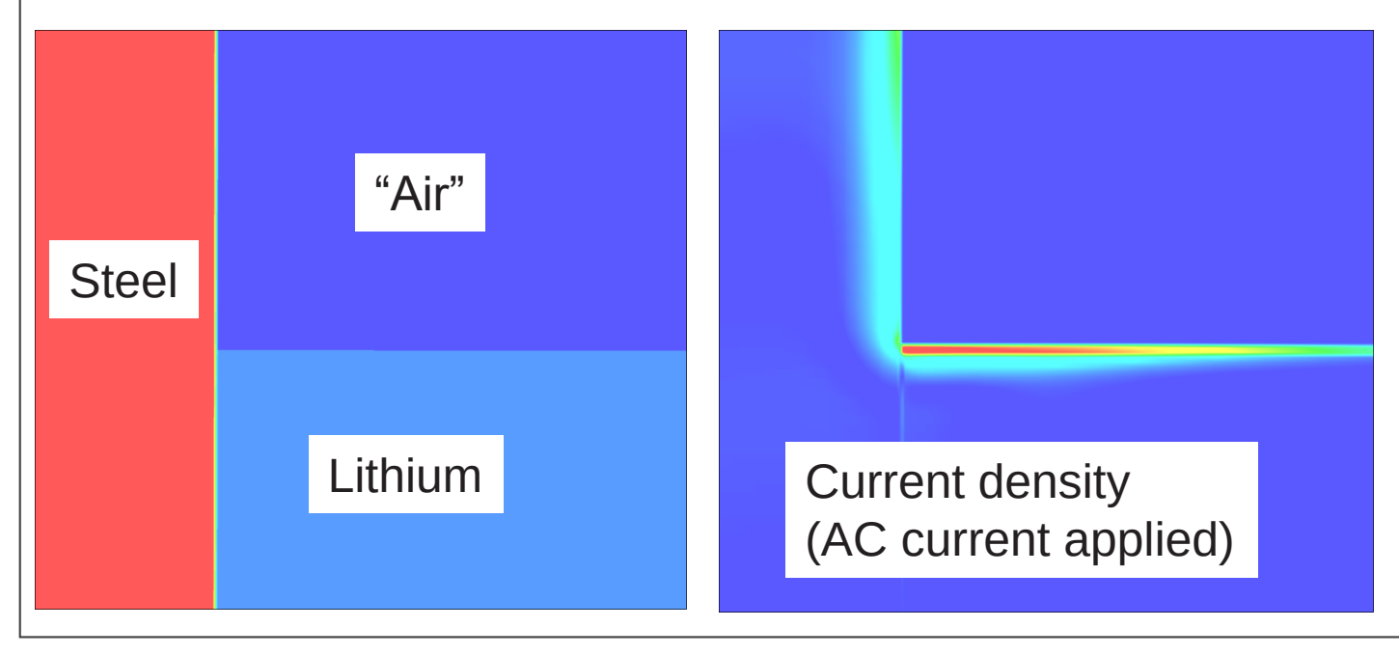
Voltage Feed



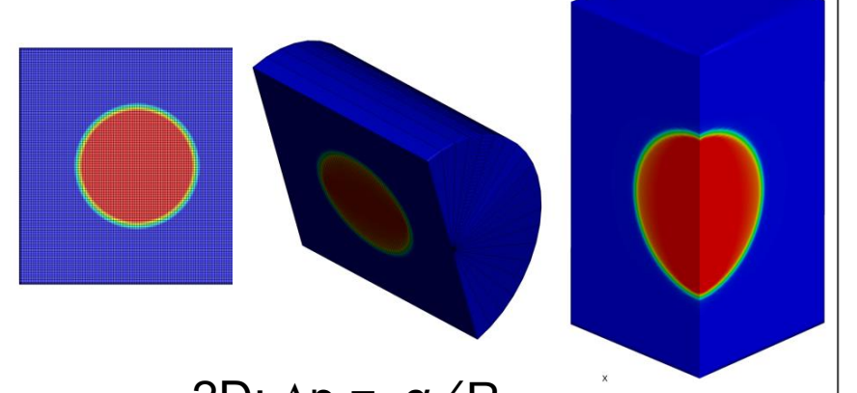
Freq, **f** = 1e+5 Hz, σ = 1e+6 S/m
Skin Depth, δ = 1.6 mm



Material discontinuities



Surface tension force calculation

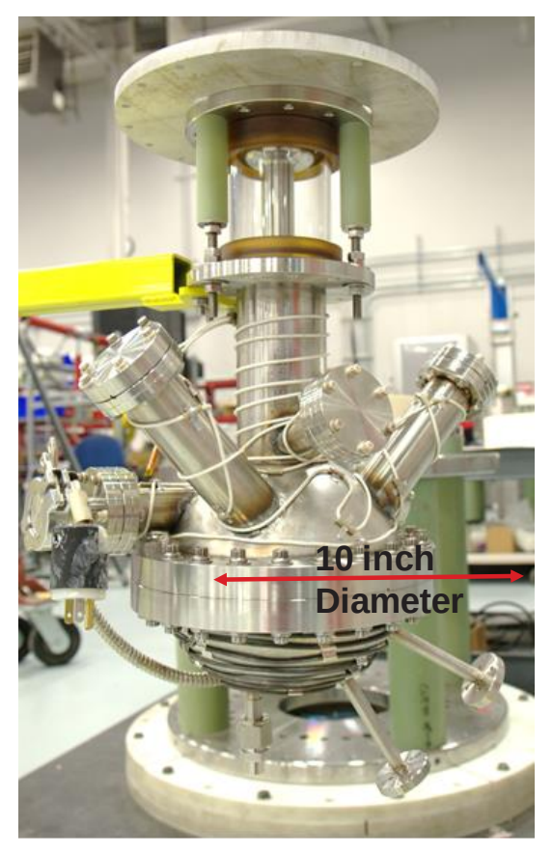


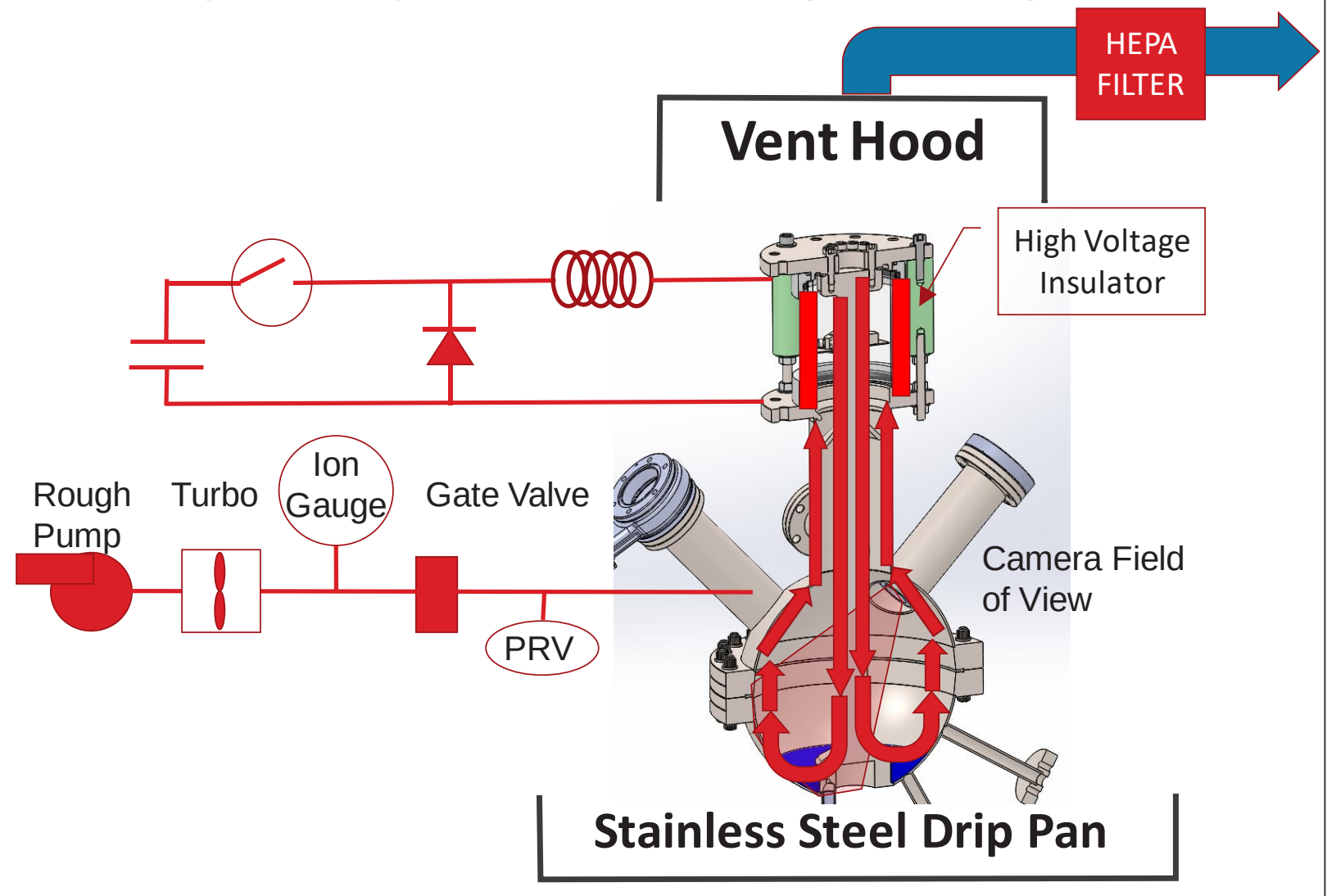
2D: Δp = α/R
3D: Δp = 2α/R

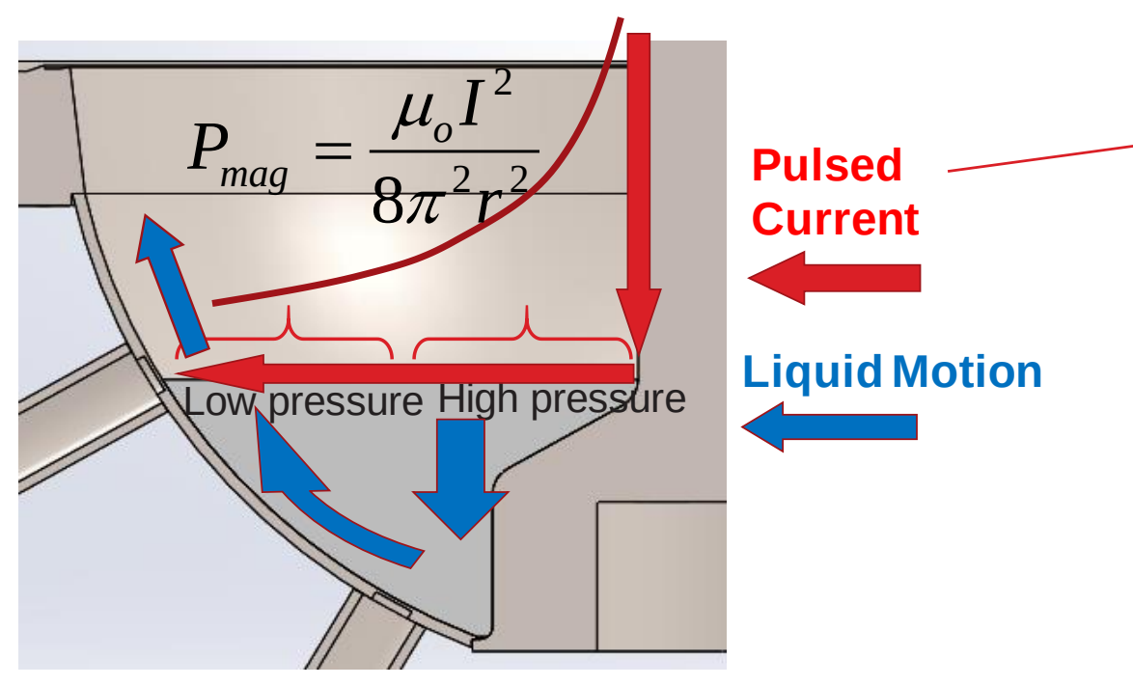
SLiC Application

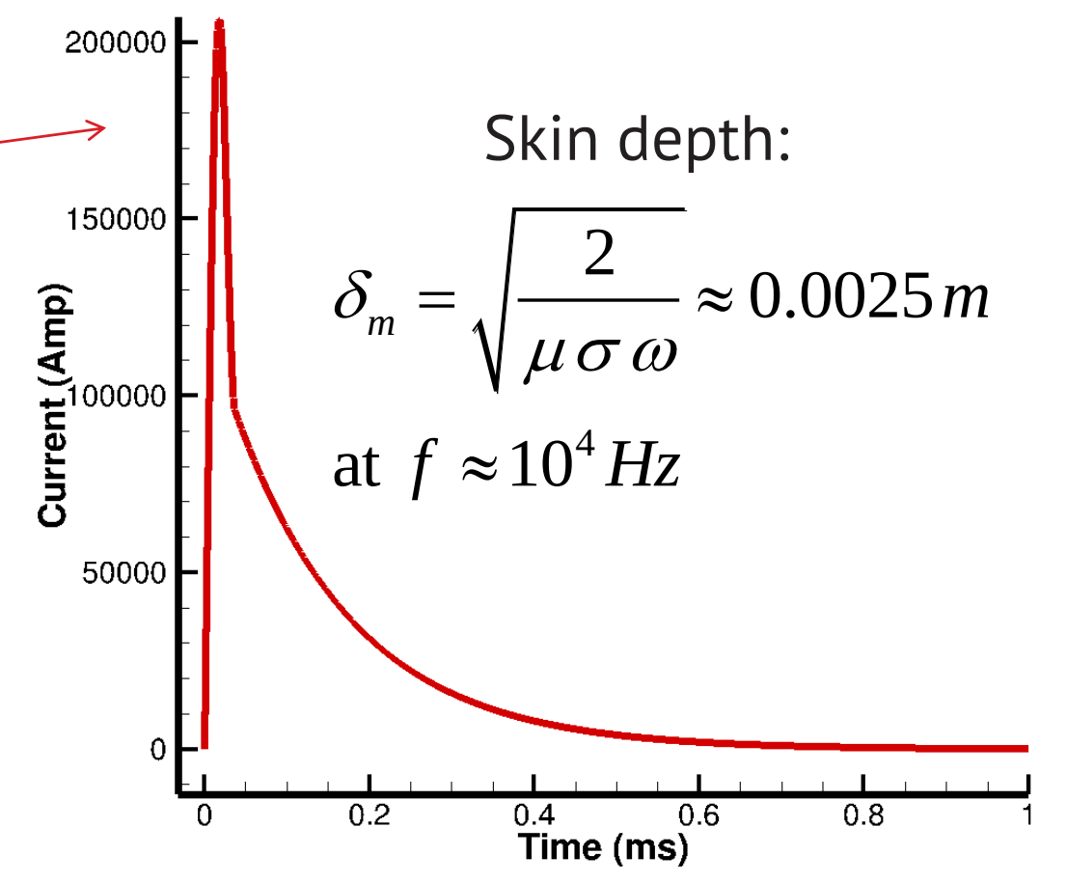
SLiC (**S**PECTOR **L**ithium **I**nverted **C**onfiguration) is a campaign to build a small scale plasma injector with a significant portion of its plasma facing wall comprised of liquid lithium. The plasma injector will be similar in scale and geometry to General Fusion's SPECTOR series of plasma experiments.

Prior to the construction of SLiC -the **mini-SLiC** experiment was built to demonstrate dynamics of liquid lithium in a sub scale SPECTOR geometry during the transient conditions of plasma formation. Mini-SLiC is used to validate MHD tools such as HIMAG. Mini-SLiC nominally applies a 25us long 200kA pulse to a coaxial geometry that contains a pool of liquid lithium.



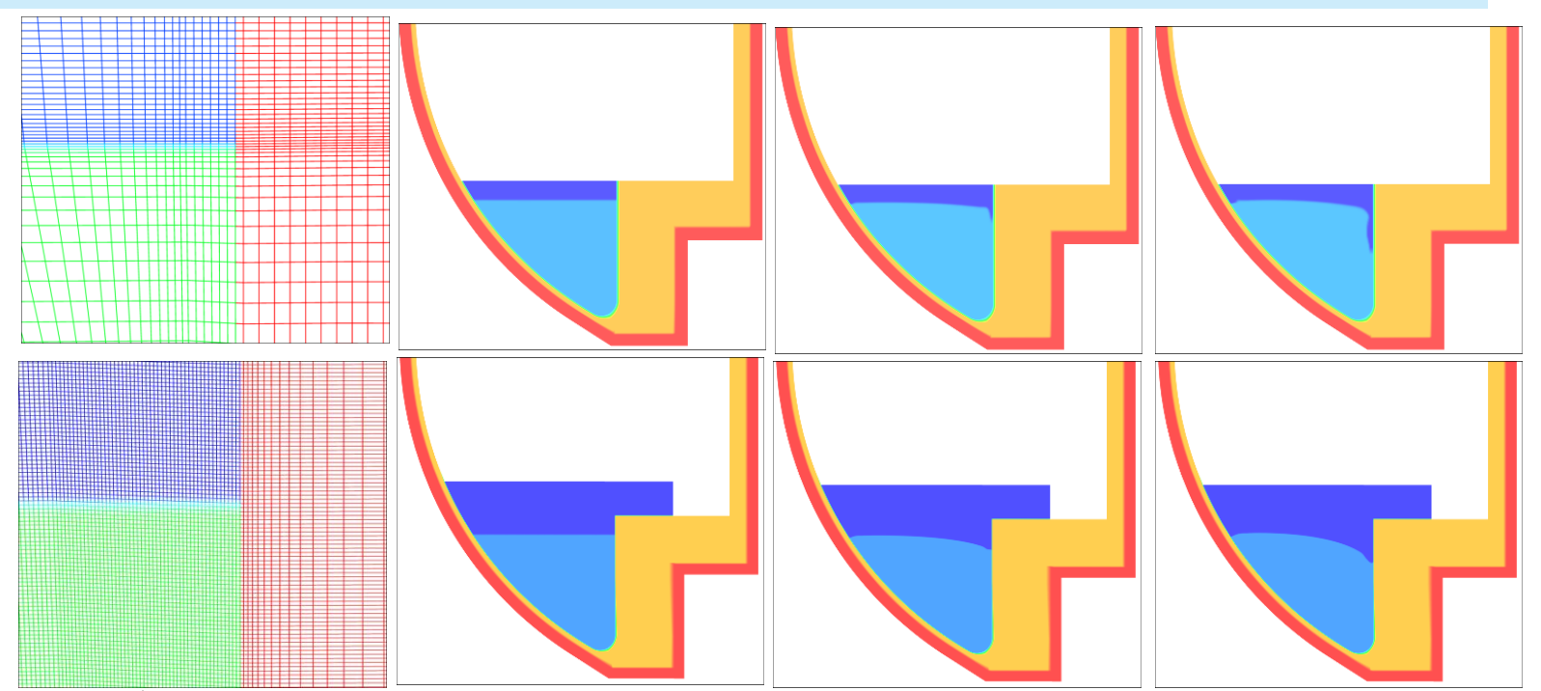


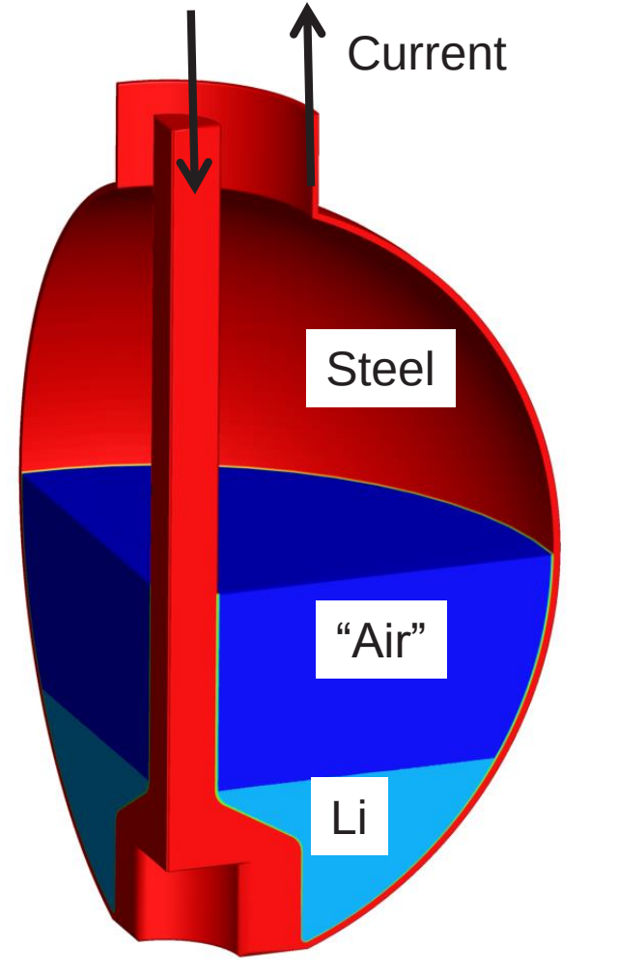




The goal of our CFD simulations was to validate the numerical models, and to find ways to make simulations accurate and efficient in unsteady axisymmetric, as well as fully three-dimensional flows

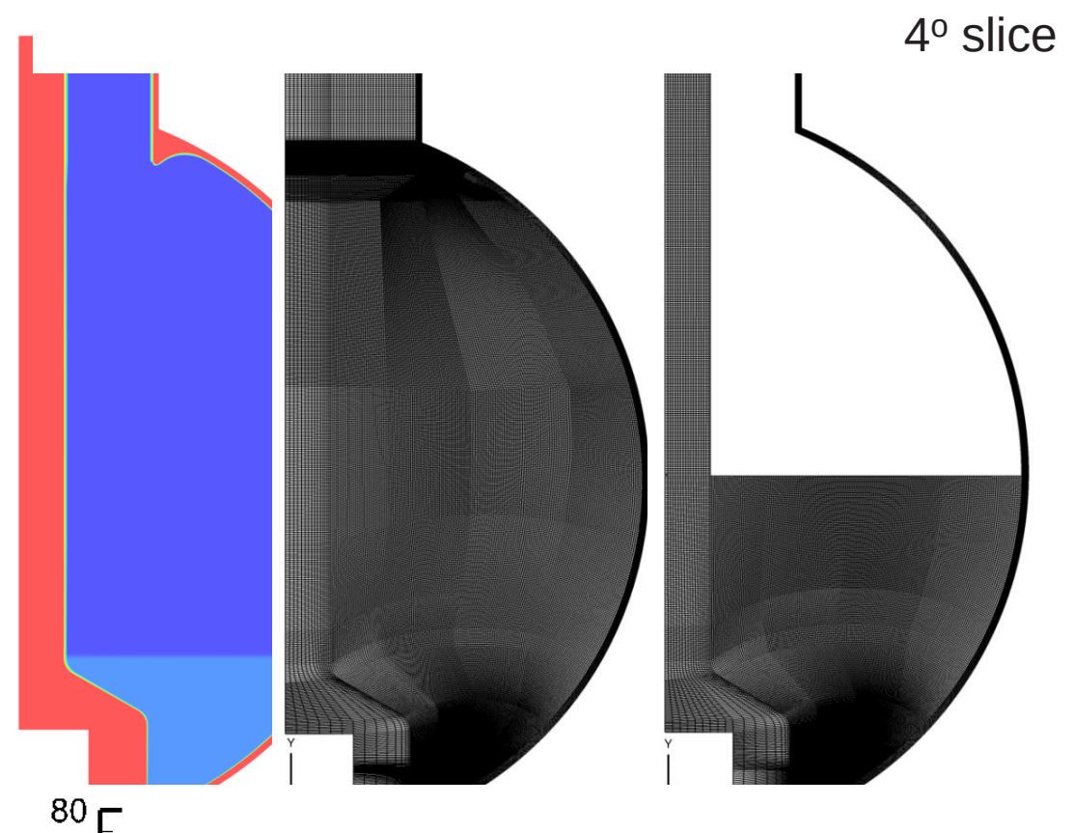
Preliminary demonstrations using the Biot-Savart model provided basic understanding of modeling and mesh related issues. Coarse and fine mesh solutions shown:

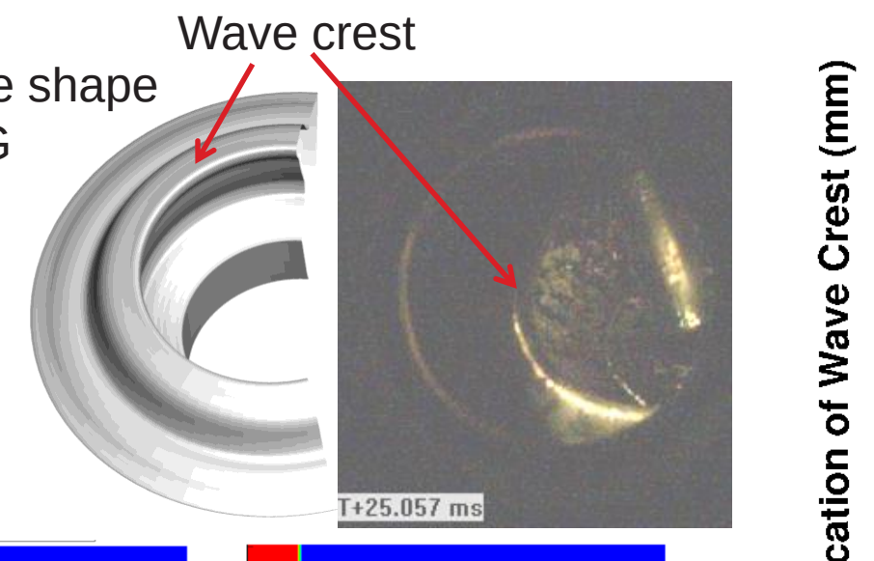




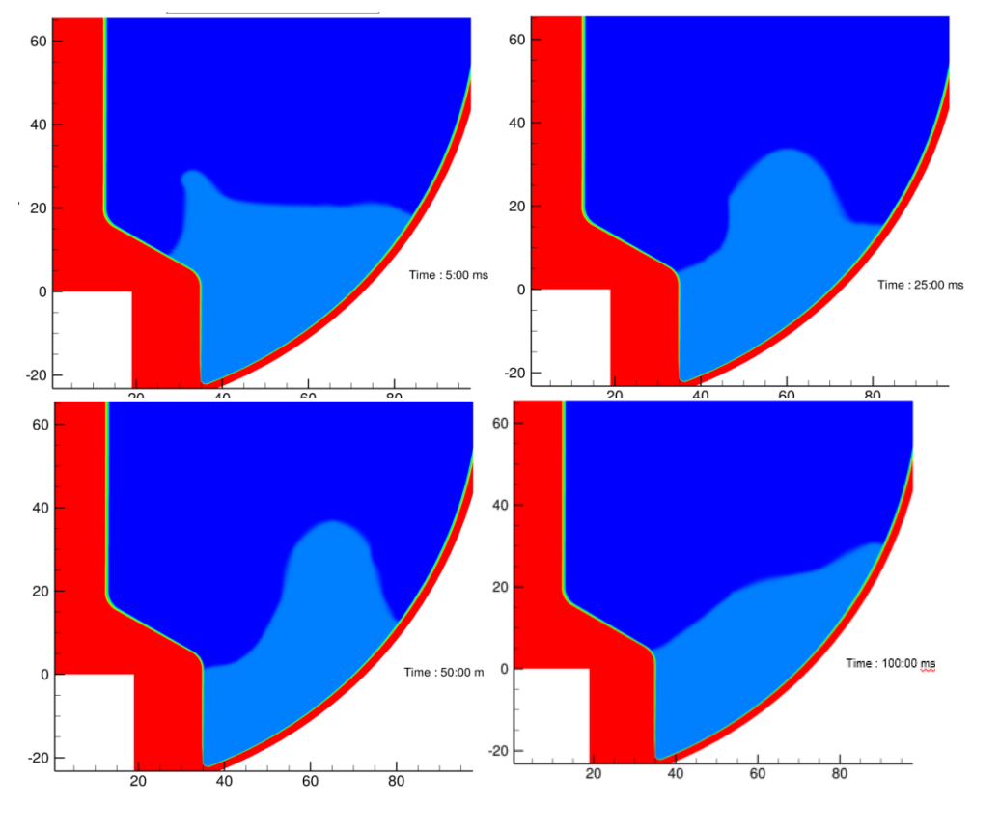
Mini SLiC CFD model: Thin (4º) slice as well as 90º and 180º chamber geometries were modeled

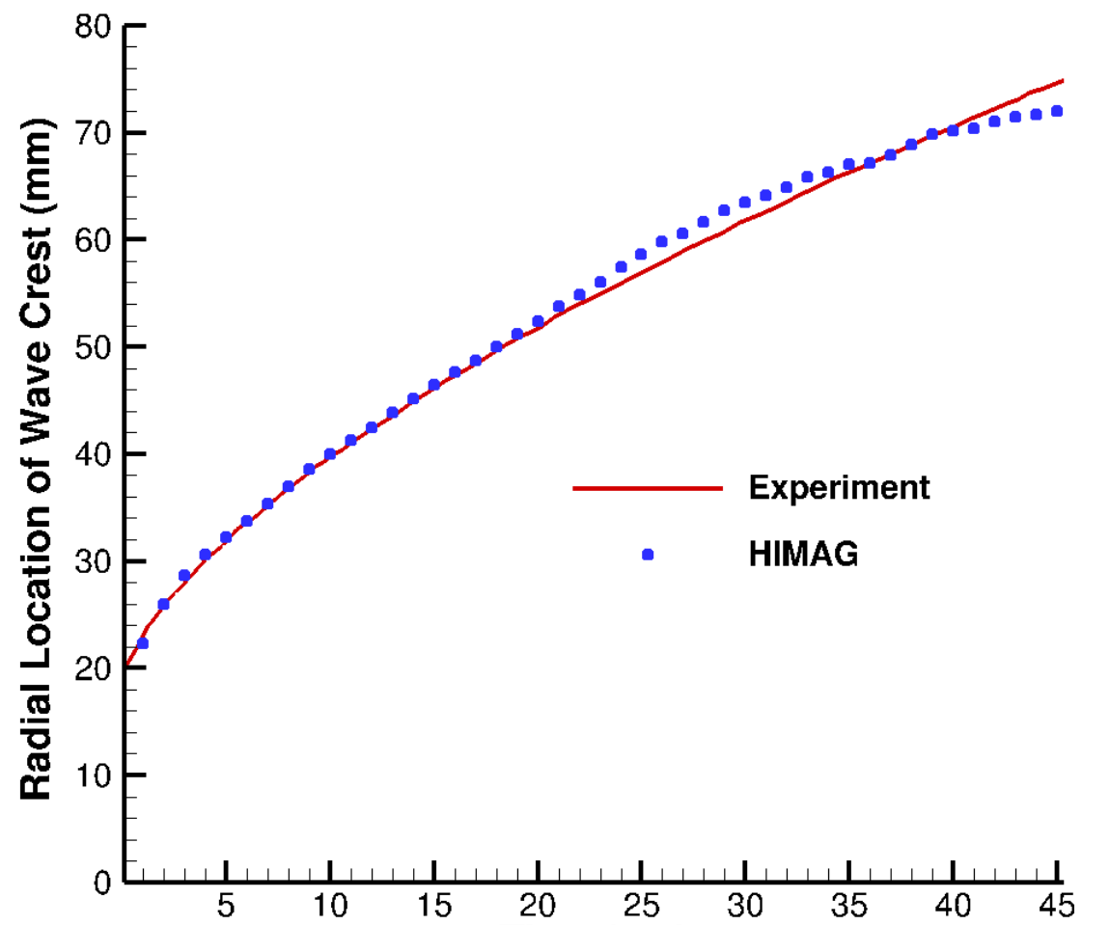
Meshes used: 55,000 and 100,000 cells for the 4º slice

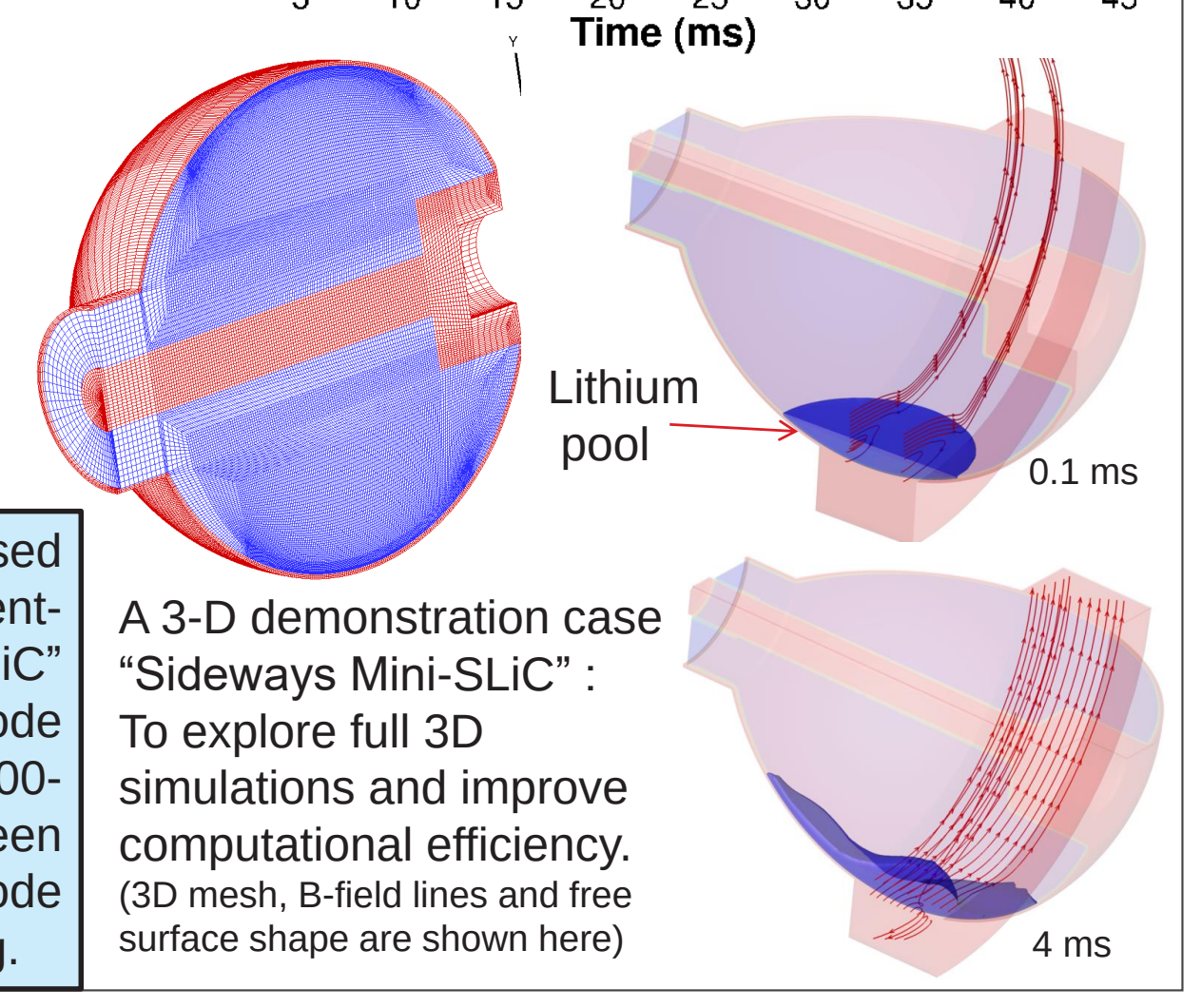




Free surface shape from HIMAG (left), and Image from experiment (right), and snapshots (below)







A 3-D demonstration case "Sideways Mini-SLiC" : To explore full 3D simulations and improve computational efficiency.
(3D mesh, B-field lines and free surface shape are shown here)

Summary: A magnetic vector potential based method was used to solve unsteady current-driven MHD flows. Data from the "mini-SLiC" experiment at General Fusion was used in code validation. Simulations were performed on 100-500 CPU cores. Promising results have been obtained so far. Further work to improve code performance in complex geometries is ongoing.