Bayesian Equilibrium Reconstruction for General Fusion Demonstration Plant

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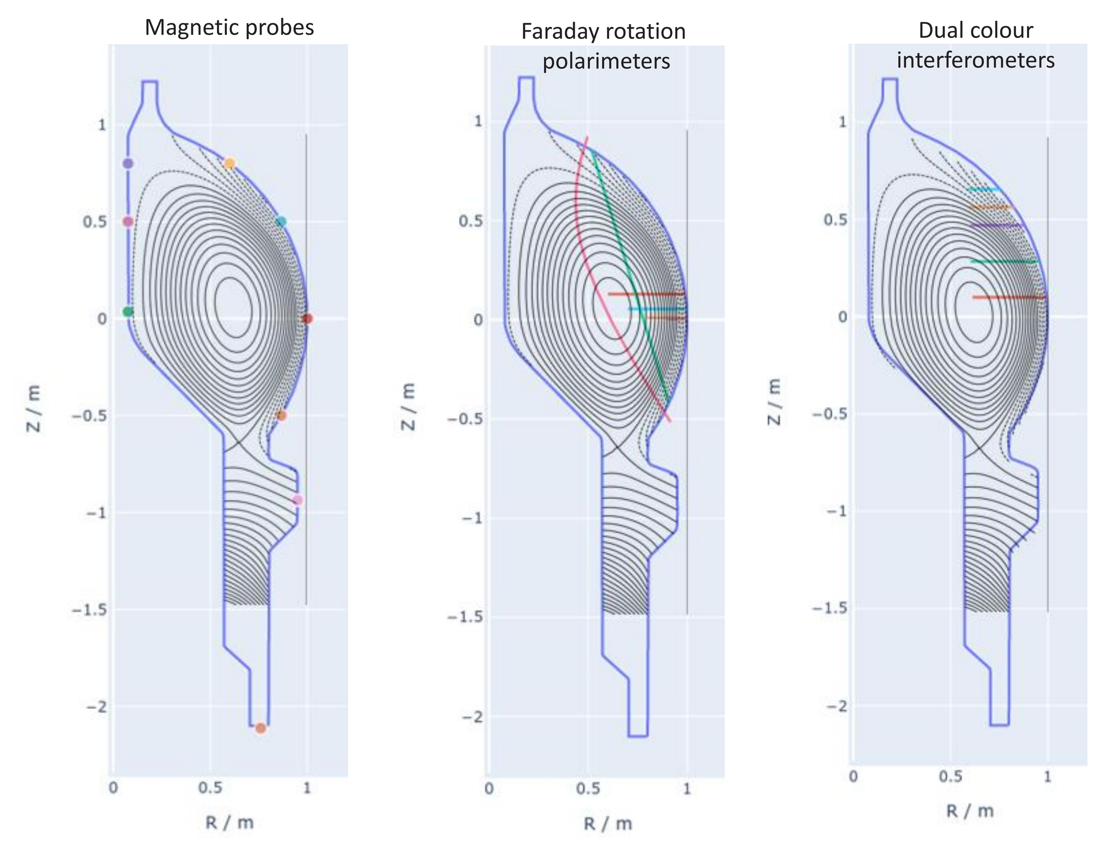
Plasma injector 3 (Pi3) at General Fusion

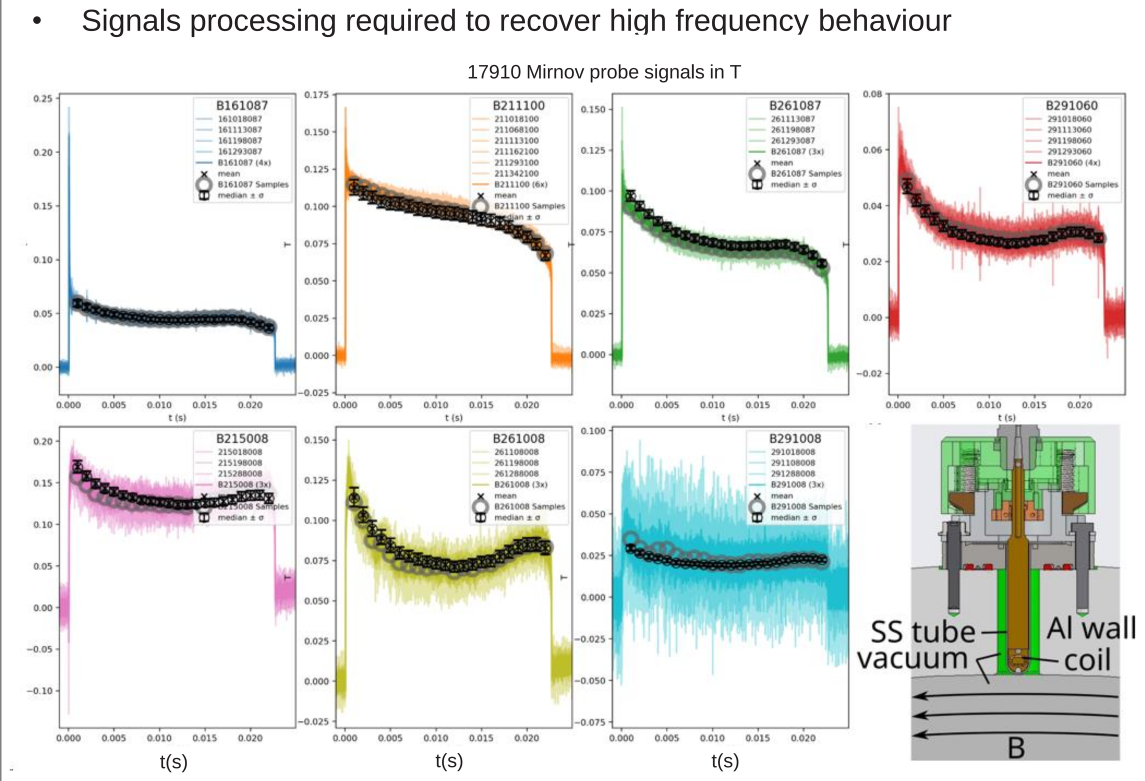
- General Fusion (GF) aims at building a magnetized target fusion power plant based on compression of a magnetically-confined plasma by liquid metal.
- Pi3 is a coaxial magnetized Marshall gun which forms a spherical tokamak plasma configuration through fast coaxial helicity injection (CHI).
- Plasma heating is accomplished through Ohmic decay of the confining magnetic field. No additional heating or current drive systems are needed.
- Pi3 will determine if the stability and confinement of the plasma are suitable for compression to fusion conditions by a fast implosion of a liquid lithium flux conserver cavity (no plasma compression in Pi3).
- Plasma reconstruction for Pi3 allows to investigate the time evolution of the main plasma parameters such as the safety factor, q, and it is, thus, crucial for the study of MHD stability

Plasma Injector 3 Diagnostic Suite



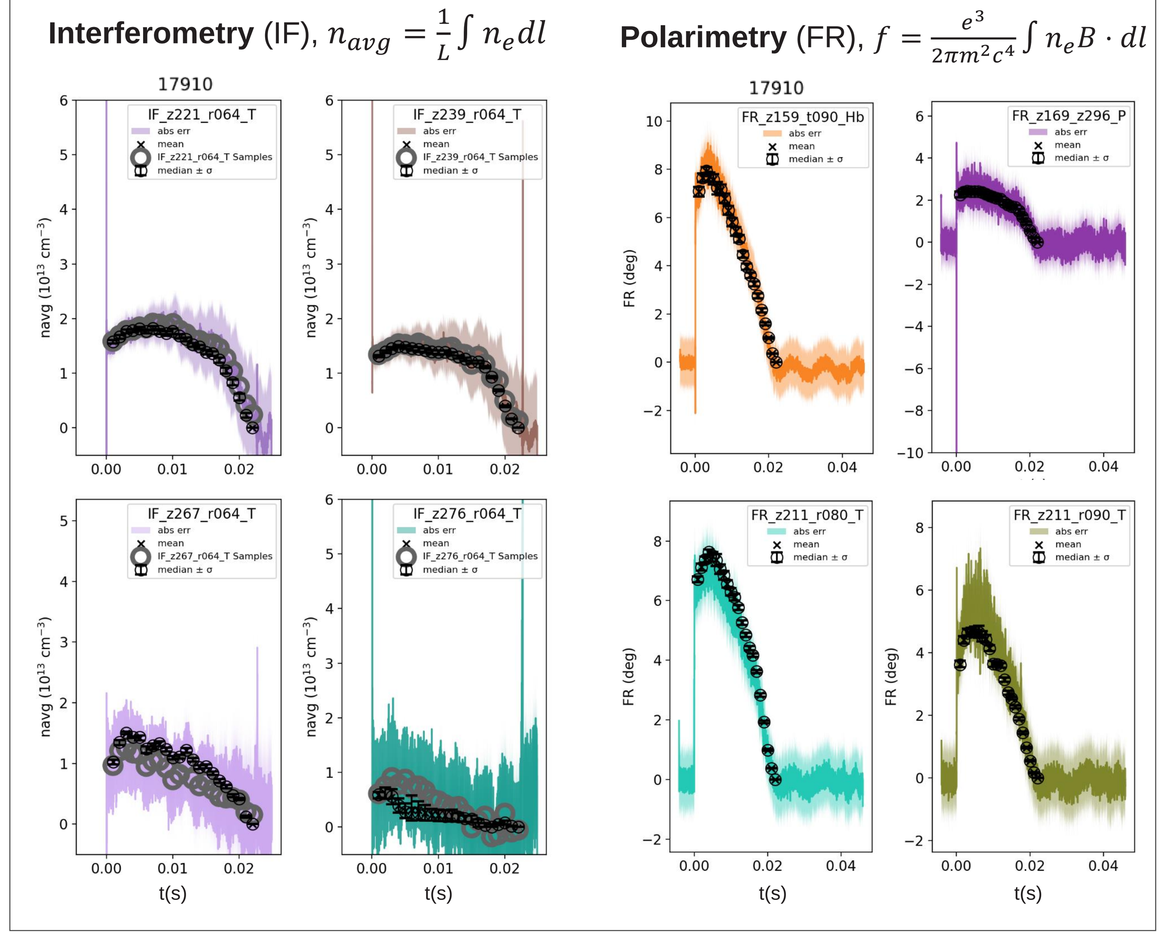
Magnetic probes in metal wall

- Mirnov probes embedded in bore holes and shielded by a thin layer of stainless steel.
- Signals processing required to recover high frequency behaviour



Laser probes

Interferometry (IF), $n_{avg} = \frac{1}{L}\int n_e dl$ **Polarimetry** (FR), $f = \frac{e^3}{2\pi m^2 c^4}\int n_e B \cdot dl$



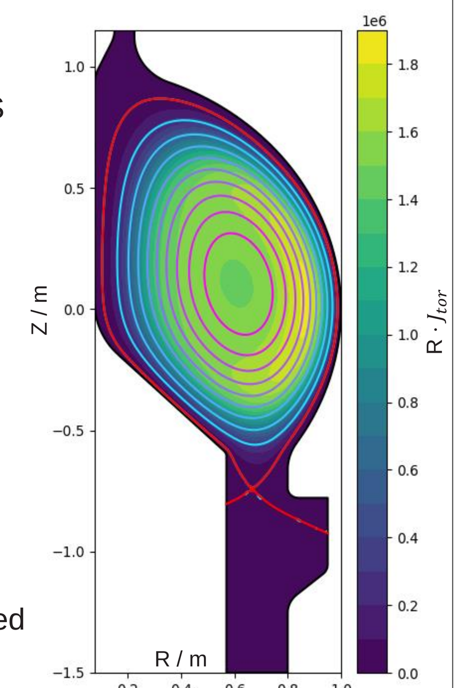
Plasma equilibrium

The Grad-Shafranov (GS) equation [1, 2] describes an axisymmetric MHD equilibrium

$$\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} = -\mu_0 r^2\frac{dp}{d\psi} - F\frac{dF}{d\psi} = -\mu_0 r^2\frac{dp}{d\psi} + \lambda(\psi)\left(\int_0^\psi \lambda(\psi_i)d\psi_i + F_{wall}\right),$$

$$\lambda(\psi) \equiv -\frac{dF}{d\psi} = -\frac{d(RB_\phi)}{d\psi}, \qquad \text{in a force-free configuration: } \lambda(\psi) = \frac{\mu_0 J\cdot B}{B^2} = \frac{|J|}{|B|}$$

FLAGSHIPS – General Fusion's GS solver



- Weak form of GS equation solved with FEniCS library

$$\int_V \frac{1}{R}\nabla\bar\Psi\cdot\nabla\chi\, dRdZ =$$
$$= \int_V \chi\left[\left(\frac{2\pi}{\Psi_{enc}}\right)^2\left(\mu_0 R\frac{\partial p}{\partial\bar\Psi} + \frac{1}{R}F\frac{\partial F}{\partial\bar\Psi}\right)\right]dRdZ,$$

where $\mu_0 J_\phi = \frac{2\pi}{\Psi_{enc}}\left(-\mu_0 R^2\frac{\partial p}{\partial\bar\Psi} - F\frac{\partial F}{\partial\bar\Psi}\right)$

- Direct solve of GS equation adjusts LCFS to externally applied fields
- Code is 90% Python with some core components in C++ and C# for speed
- Domain covers vacuum region with unstructured mesh
- The 3 most observable parameters with external magnetic probes are
  - Plasma current, I_p
  - Internal inductance, ℓ_i
  - Poloidal beta, β_p
- For circular cross-section, β_p and ℓ_i are combined in $\Lambda \equiv \beta_p + \ell_i/2$
- A proxy for internal inductance is the lambda centroid, c, that can be computed without solving the GS equation

$$c = \frac{\int\bar\psi\lambda(\bar\psi)d\bar\psi}{\int\lambda(\bar\psi)d\bar\psi}.$$

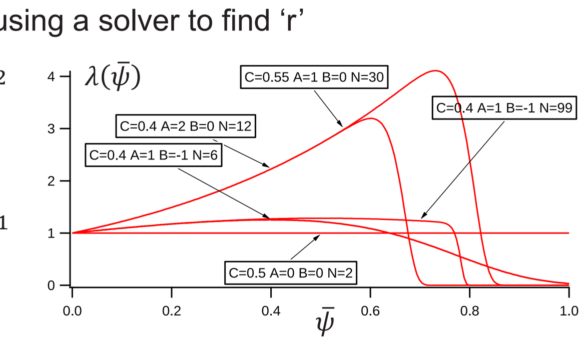
- FR polarimetry chords help constrain internal current density profile
- Interferometry chords help constrain plasma density profile

Inputs to FLAGSHIPS

- ψ boundary conditions on the vessel wall determined from DC coil current modelled in FEMM matched against flux loop measurements, or approximate calculations based on the history of B-poloidal on the wall
- Current density profile, modelled with the Nevins lambda

$$\lambda(\bar\psi) = \lambda_0\exp(a\bar\psi + b\bar\psi^2 - r\bar\psi^n),$$

we specify centroid c instead of 'r', using a solver to find 'r'



- Pressure profile, $p(\psi) = p_0(1-\psi^2)^2$
- Constraints on I_p or on closed poloidal flux
- Constraints on pressure set by β_{pol1}
- Current ratio I_p/I_{shaft}

Look-up tables (LUT) of pre-calculated equilibria

A 7-dimension LUT is created with axes:

- Plasma current over shaft plasma, I_p/I_{shaft}
- Poloidal beta, $\beta_{p1} = 2\mu_0 p_{va}/<B_{pol}>^2_{sa}$
- Lambda centroid, c
- Nevins parameters, a, b, n
- Magnitude of magnetic diffusion at wall (spatial profile fixed) normalized by I_{shaft}
- Magnitude of vacuum flux at wall (spatial profile fixed) normalized by I_{shaft}

Each table contains 1-3 million entries

Bayesian Reconstruction

Bayes theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- A: given equilibrium is "truth"
- B: measurement recorded
- P(A) is uniform

$$P(B|A) = \exp\left(-\frac{1}{2}\sum_{probe}\left(\frac{m'_i - m_i}{\sigma_i}\right)^2\right)$$

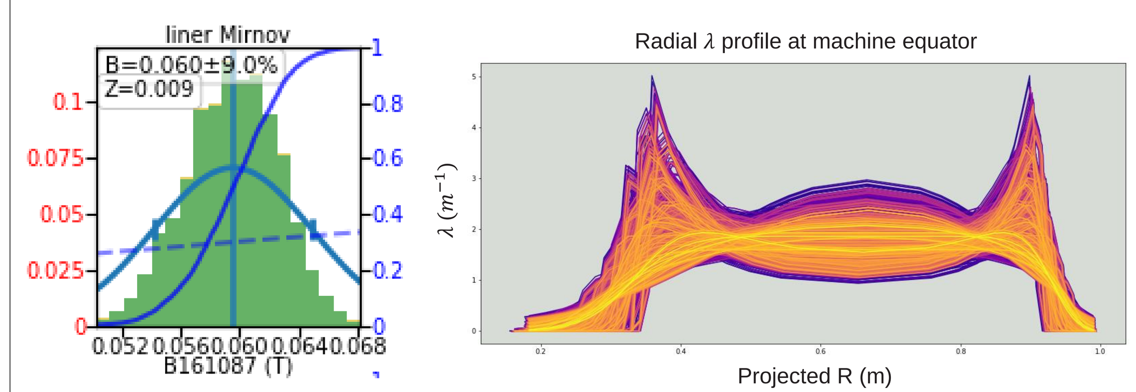
where m'_i and m_i are recorded and synthetic measurements for the i-th probe

- P(B) chosen to set $\sum P(A|B) = 1$

$P(A|B)$ is computed for each equilibrium in the LUT (+ interpolation) and probability density functions (PDF) are calculated and plotted as histograms for any scalar property value.

WARNING: the normalization assumes that the LUT covers all possible equilibria

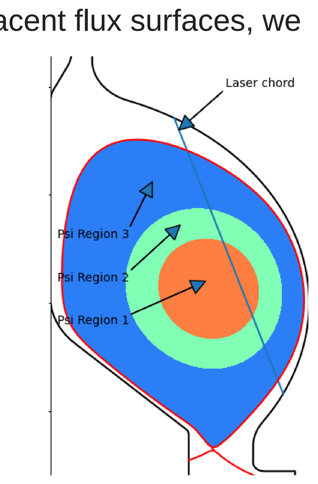
- Measurement uncertainties are shown as Gaussian curves
- Prior and posterior cumulative density functions (CDF) are shown as dashed and solid blue lines respectively
- When prior and posterior CDFs are similar, we know nothing about that quantity
- For safety factor and lambda a smattering of profiles within the 1-sigma group is shown
- Yellow lines are more probable profiles, whereas the blurring indicates the uncertainties



Using density for reconstruction

An equilibrium is generated from an assumed pressure profile, and hence it does not constrain density. We assume density is a piecewise linear function of ψ. For $\psi_i < \psi < \psi_{i=1}$, with ψ_i and ψ_{i+1} any pair of adjacent flux surfaces, we write the density as

$$n(\psi) = n(\psi_i) + \left(n(\psi_{i+1}) - n(\psi_i)\right)\frac{\psi - \psi_i}{\psi_{i+1} - \psi_i},$$



in order to calculate the synthetic interferometric measurement along a chord of length L_m

$$n_{avg,s} = \frac{1}{L_m}\int_0^{LCFS} n(\psi)dl = \sum_i n_{i,i+1},$$

where

$$n_{i,i+1} = \frac{1}{L_m}\int_{\psi_i}^{\psi_{i+1}} n(\psi)dl =$$
$$= \underbrace{\left(\frac{n(\psi_i)\psi_{i+1} - n(\psi_{i+1})\psi_i}{\psi_{i+1} - \psi_i}\right)}_{\text{Chosen model}}\underbrace{\frac{1}{L_m}\int_{\psi_i}^{\psi_{i+1}} dl}_{\text{Pre-calculated (from A)}} + \underbrace{\left(\frac{n(\psi_{i+1}) - n(\psi_i)}{\psi_{i+1} - \psi_i}\right)}_{\text{Chosen model}}\underbrace{\frac{1}{L_m}\int_{\psi_i}^{\psi_{i+1}} \psi dl}_{\text{Pre-calculated (from A)}}.$$

Similarly, synthetic polarimetry measurements can be written as

$$f_{i,i+1} = \frac{e^3}{2\pi m^2 c^4}\int_{\psi_i}^{\psi_{i+1}} n(\psi)B\cdot dl =$$
$$= \frac{e^3}{2\pi m^2 c^4}\left[\underbrace{\left(\frac{n(\psi_i)\psi_{i+1} - n(\psi_{i+1})\psi_i}{\psi_{i+1} - \psi_i}\right)}_{\text{Chosen model}}\underbrace{\int_{\psi_i}^{\psi_{i+1}} B\cdot dl}_{\text{Pre-calculated (from A)}} + \underbrace{\left(\frac{n(\psi_{i+1}) - n(\psi_i)}{\psi_{i+1} - \psi_i}\right)}_{\text{Chosen model}}\underbrace{\int_{\psi_i}^{\psi_{i+1}} \psi B\cdot dl}_{\text{Pre-calculated (from A)}}\right].$$

If we define a vector with all free parameters, such as for instance $w = [k, n_1, n_2]^T$, we compute the synthetic measurements from the equilibrium A as

$$m = Aw,$$

where m includes as many measurements as the diagnostics included in A.

The probability of a measurement set m' matching the synthetic set m obtained through A is

$$P(m'|A) = \exp\left(-\frac{1}{2}\sum_{probe}\left(\frac{m'_i - m_i}{\sigma_i}\right)^2\right),$$

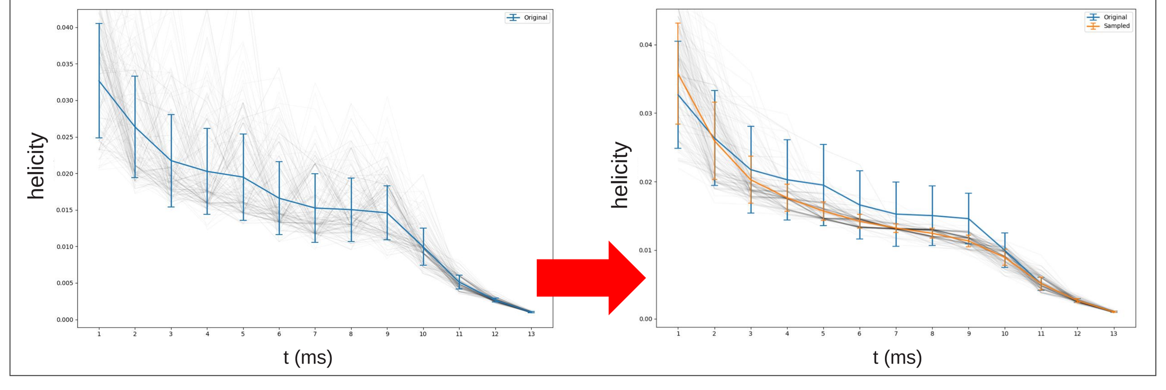
which can be cast as $P(m'|A) = \exp\left(-\frac{1}{2}(m' - Aw)^T C^{-1}(m' - Aw)\right)$.

Solving for the vector w^* that minimizes the error one finds

$$w^* = (A^T C^{-1} A)^{-1} A^T C^{-1} m'.$$

Helicity time constraint

- Original Bayesian reconstruction is time-independent
- We apply time constraints by sampling trajectories of equilibria according to the posterior PDFs of each time slice
- Currently we apply a single binary constraint: helicity must not increase

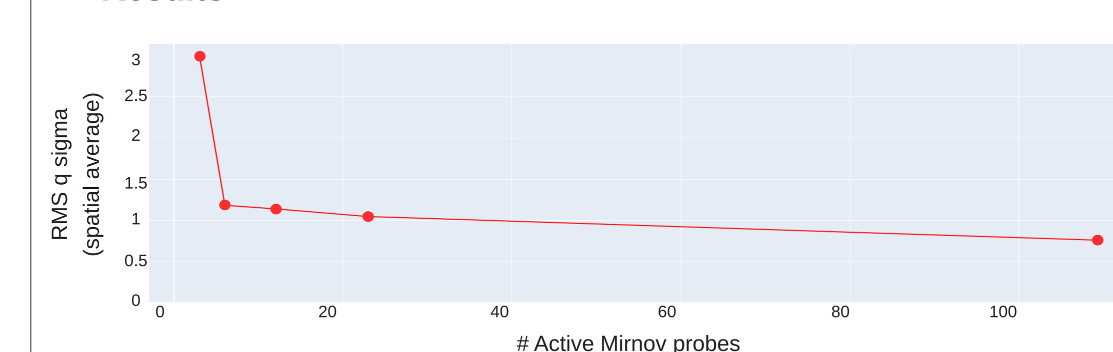


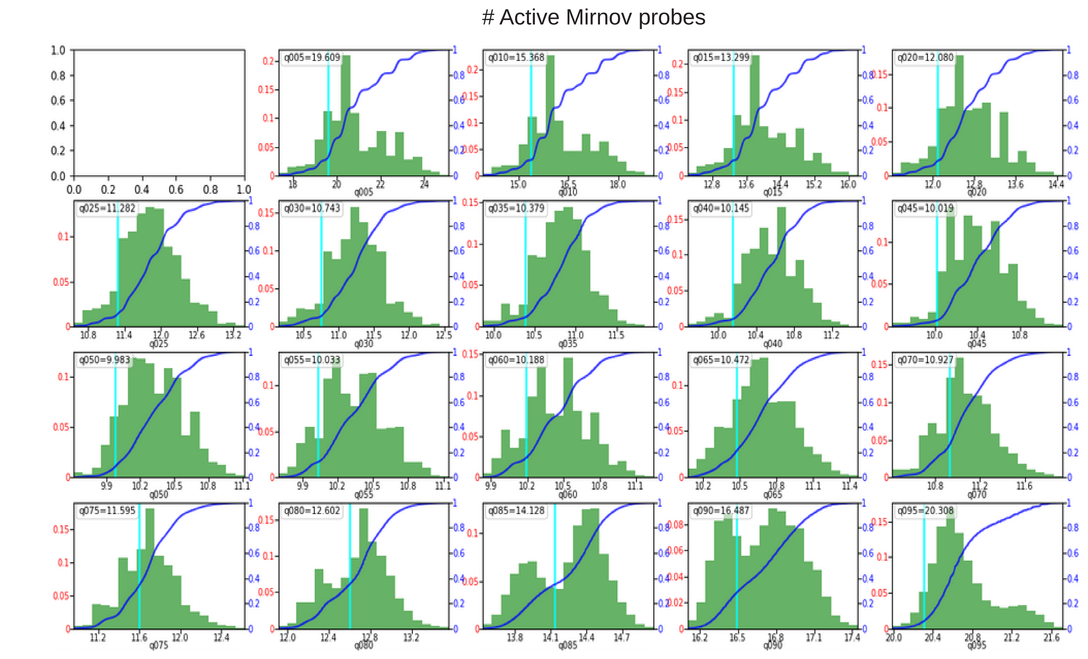
Sensitivity Analysis

Introduction

- Sensitivity analysis estimates the accuracy and precision of the reconstructed values (e.g. q profile) for a given set of sensors and measurements
- This is used to determine several things:
  - Required sensors and uncertainties to achieve target accuracy/precision in reconstructed values
  - Optimal configuration for limited numbers of sensors
  - Cost/benefit analysis of expensive diagnostics
- Testcases: synthetic measurement data generated by post-processing magnetic simulations (GS equilibria, VAC)
- Synthetic density measurements are calculated using the magnetic simulations and assumed density profiles
- Errors can be easily calculated as the true values of the reconstructed quantities are known
- Errors can be compared to targets or used to select optimal sensor configurations

Results





- Histograms showing the probability distribution for the q at the 5% $\bar\psi$ intervals
- Cyan bars show true values from testcase data

Limitations

- Phenomena missing from simulation codes will not be reflected in the results.
- Sensitivity analysis uses a set of testcases, which may not be representative of the experimental plasmas (on Pi3, the 1000 most experimentally common profiles are used, while on FDP the LUT is uniformly sampled).
- Reconstruction is computationally expensive and slow. Brute force optimization often requires too many combinations to be feasible.

Future improvements

- **Markov chain Monte Carlo method**
  - Initialize with sparse table
  - Calculate steps until method converges to equilibrium
  - Can get stuck in local minima for long time
  - Requires on-the-fly equilibrium calculations
- **Inverse equilibrium**
  - Much faster
  - Requires experimental determination of LCFS
- **Adding presumed temperature profile $T(\psi)$ and neoclassical resistivity would allow to compute the time derivatives of all synthetic probes**
  - This doubles the measurements information we are leveraging
  - Using Bayesian inference, we can then calculate the probabilities of the various temperature profiles
  - Confine equilibrium selection. . Some lambda profiles might fit the measured probe values, but not be able to produce the measured slopes with a physically realistic temperature profile.

References

[1] Grad, H., and Rubin, H. (1958) Proc. 2nd UN Conf. Peaceful Atomic Energy, Vol. 31, Geneva: IAEA p. 190.
[2] Shafranov, V.D. (1966) Reviews of Plasma Physics, Vol. 2, New York: Consultants Bureau,p.103.